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COMPUTER PROGRAM FOR RELAXATION SOLUTIONS OF THE NONLINEAR SMALL-DISTURBANCE EQUATIONS FOR TRANSONIC FLOW IN AN AXIAL COMPRESSOR BLADE ROW

William J. Rae

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This report contains a description of a computer program for calculating the three-dimensional transonic flow through a compressor blade row, in the nonlinear small-distrubance approximation, and for subsonic values of the inlet relative Mach number at the blade tip.

The problem formulation is reviewed briefly; following this, a description of the program is given together with a listing and sample case.

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ABSTRACT

This report contains a description of a computer program for calculating the three-dimensional transonic flow through a compressor blade row, in the nonlinear small-disturbance approximation, and for subsonic values of the inlet relative Mach number at the blade tip.

The problem formulation is reviewed briefly; following this, a description of the program is given together with a listing and sample case.

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INTRODUCTION

Three-dimensional transonic flows through a compressor blade row represent a very complex problem. Some of the complications that make the problem very difficult to solve are the presence of adjacent zones of subsonic and supersonic flow, and the communication between these zones that takes place through radial pressure gradients.

As a first step in applying recent computational techniques to this problem, a study of the nonlinear small-disturbance approximation has been made[1-4]. This report contains a description of the computer program used to obtain the numerical results presented in these papers. For convenience, the basic equations used are summarized in Section 1. The program itself is described in Section 2, and a sample case is discussed in Section 3. A listing of the program and a guide to preparing the input are given in the Appendices.

The finite-difference equations used are the set on which the results of Reference 4 were based. They supersede, in a few small details, the earlier sets used in References 1-3.

It should be stressed that the program cannot handle cases where the inlet relative Mach number is supersonic at the tip, because the farfield boundary condition used is not a radiation condition. Thus, Mach waves, originating at the blade row, reflect from the grid boundaries instead of escaping as they should. As a result, the solution will contain a set of waves which pass back and forth between the upstream and downstream edges of the grid.

It should also be pointed out that some of the program options have not been used extensively. In particular, only a few runs have been made in the design mode, where the loading and thickness distributions are prescribed. For this reason, there are unknown limits on the usable ranges for some of the parameters such as step size and relaxation factors.

Section 1

SUMMARY OF BASIC EQUATIONS

The coordinate system and rotor geometry used are shown in Figure 1, where the dimensionless variables are defined as

$$z = \frac{\omega x}{U_\infty} \quad , \quad \rho = \frac{\omega r}{U_\infty} \quad , \quad \zeta = \theta - z \quad ; \quad \phi = \frac{\omega \varphi}{U_\infty^2} \tag{1}$$

The velocity components seen by a blade-fixed observer are

$$W_x = U_\infty + u \quad , \quad W_r = v \quad , \quad W_\theta = \omega r + w \tag{2}$$

These dimensional perturbation velocities are related to the velocity potential by

$$\begin{aligned}
\bar{u} &= \frac{u}{U_\infty} = \left.\frac{\partial \phi}{\partial z}\right)_{\rho,\theta} = \left.\frac{\partial \phi}{\partial z}\right)_{\rho,\zeta} - \left.\frac{\partial \phi}{\partial \zeta}\right)_{z,\rho} \; ; \\
\bar{v} &= \frac{v}{U_\infty} = \left.\frac{\partial \phi}{\partial \rho}\right)_{z,\theta} = \left.\frac{\partial \phi}{\partial \rho}\right)_{z,\zeta} \\
\bar{w} &= \frac{w}{U_\infty} = \frac{1}{\rho}\left.\frac{\partial \phi}{\partial \theta}\right)_{z,\rho} = \frac{1}{\rho}\left.\frac{\partial \phi}{\partial \zeta}\right)_{z,\rho}
\end{aligned} \tag{3}$$

The velocity potential satisfies the equation

$$\begin{aligned}
&\left\{1 - M_\infty^2(1+\rho^2) - (\gamma+1)M_\infty^2 \phi_z\right\}\phi_{zz} + \rho^2\phi_{zz} - 2(1+\rho^2)\phi_{\zeta z} \\
&\quad + \frac{(1+\rho^2)^2}{\rho^2}\phi_{\zeta\zeta} + (1+\rho^2)\left(\phi_{\rho\rho} + \frac{1}{\rho}\phi_\rho\right) = 0
\end{aligned} \tag{4}$$

Subscripts denote derivatives in the z, ρ, ζ coordinate system; thus the symbol ϕ_z stands for $\left.\frac{\partial \phi}{\partial z}\right)_{\rho,\zeta}$. There are B blades in the row, and they are taken to lie in the helical surfaces defined by U_∞ and ωr. The axial projection of their chord is a constant, c_a. Thus, they are located at

$$0 \le z \le \frac{\omega c_a}{U_\infty} \; ; \; \rho_H \le \rho \le \rho_T \; ; \; \zeta = \frac{2j\pi}{B} \; , \; j = 0, 1, 2, \ldots, B-1 \tag{5}$$

If the program is run in the off-design mode, the blade shapes are given by (see Figure 2)

$$n_{\substack{u\\L}}(s,r) = h(s,r) \pm \frac{1}{2}\, t(s,r) - \alpha(r)\cdot s \tag{6}$$

where h, t and α are the camber, thickness, and angle of incidence. The blade-surface boundary condition is

$$\frac{u_n}{W_o} = \frac{dn}{ds} \tag{7}$$

When the program is run in the design mode, the prescribed quantities are the loading and thickness distributions:

$$\Delta C_p(s,r) \equiv \frac{p_L - p_u}{\frac{1}{2}\,\rho_\infty\, u_\infty^2} = -2\,(1+\rho^2)\left[\left.\frac{u_s}{W_o}\right)_L - \left.\frac{u_s}{W_o}\right)_u\right] \tag{8}$$

$$t'(s,r) = \frac{dn_u}{ds} - \frac{dn_L}{ds} \tag{9}$$

The blade-surface boundary conditions are applied in the helical surfaces $\zeta = 0$ and $\zeta = 2\pi/B$.

Far upstream of the blades, the perturbation potential is set equal to zero; far downstream, it is expressed as $\phi = Cz$, where

$$C = \frac{-B}{\pi\,(1-M_\infty^2)(\rho_T^2-\rho_H^2)} \int_{\rho_H}^{\rho_T} \rho\,\Delta\phi\; d\rho \tag{10}$$

Section 2

PROGRAM DESCRIPTION

A guide to the preparation of the input is given in Appendix A, while the listing of the program itself and a dictionary of the FORTRAN variables are given in appendices B and C.

The program begins by reading and printing input values. The finite-difference grid is then calculated, in subroutine GRID. The region in which the solution is to be found is divided into a grid, with the indices L, K and N used to number points in the ζ, z and ρ directions, respectively. Equal spacing is used in the ζ- and ρ- directions:

$$\zeta(L) = (L-1)\Delta\zeta \ , \quad L = 1, 2, \ldots, LMX \ ; \quad \Delta\zeta = \frac{2\pi}{B} \Big/ (LMX-1) \tag{11}$$

$$\rho(N) = \rho_H + (N-1)\Delta\rho \ , \quad N = 1, 2, \ldots, NMX \ ; \quad \Delta\rho = \frac{\rho_T - \rho_H}{NMX-1} \tag{12}$$

The spacing in the z- direction is nonuniform; the z- coordinate is taken as

$$z = z(\tau) \quad ; \quad \frac{\partial}{\partial z} = f\frac{\partial}{\partial \tau} \quad , \quad f \equiv \frac{d\tau}{dz} \tag{13}$$

The variable τ is then allowed to vary from -1 to +1, as z varied from $-\infty$ to $+\infty$.

$$\tau = -1 + K\Delta\tau \quad , \quad K = 1, 2, \ldots, KMX \ ; \quad \Delta\tau = \frac{2}{KMX+1} \tag{14}$$

The particular dependence $z(\tau)$ used here is

$$z(K) = z_M + \frac{1}{2\alpha_1} \ln \frac{1+\tau(K)}{1-\tau(K)} \tag{15}$$

where

$$z_M = \frac{1}{2}(z_I + z_B)$$

$$z_I = FXI \cdot \frac{\omega c_a}{U_\infty} \quad , \quad z_B = FXB \cdot \frac{\omega c_a}{U_\infty} \tag{16}$$

$$2\alpha_1 = \frac{\ln\left[\Delta\tau/(2-\Delta\tau)\right]}{\frac{1}{2}(z_B - z_I)}$$

The locations of the upstream and downstream edges of the grid are set by the input values for FXB and FXI.

Next, the blade-surface boundary conditions are calculated, in subroutine BVAL. These conditions depend on IBC: for IBC = 1 (the off-design case), the blade-shape parameters in the array BV are used to calculate the surface slopes DNDS on the suction and pressure sides, at each of the axial and radial grid points on the blades.

For IBC = 2 (the design case) the assigned loading and thickness distributions are used to generate the following quantities, which are used in applying blade-surface boundary conditions at L = 1 and L = LMX, respectively:

$$DNDS(KB, N, 1) = \int_0^z \frac{\Delta C_p}{2} \, dz \tag{17}$$

$$DNDS(KB, N, 2) = \frac{\rho^2}{1+\rho^2} \frac{\Delta C_p}{2} + \rho t'(s) \tag{18}$$

The program version listed here uses a parabolic-arc blade shape, for IBC = 1:

$$t(s, r) = \frac{4 t_{MAX}}{[c(r)]^2} \left\{ s \, [c(r) - s] \right\} \tag{19}$$

$$h(s, r) = \frac{4 h_{MAX}}{[c(r)]^2} \left\{ s \, [c(r) - s] \right\} \tag{20}$$

The maximum thickness and camber, which occur at $s/c = 1/2$ for this shape, are allowed to vary in the radial direction according to:

$$t_{MAX} = C_a \left\{ BV(1) + BV(2) \frac{\rho_T}{\rho} + BV(3) \frac{\rho}{\rho_T} \right\} \tag{21}$$

$$h_{MAX} = C_a \left\{ BV(4) + BV(5)\frac{\rho_T}{\rho} + BV(6)\frac{\rho}{\rho_T} \right\} \qquad (22)$$

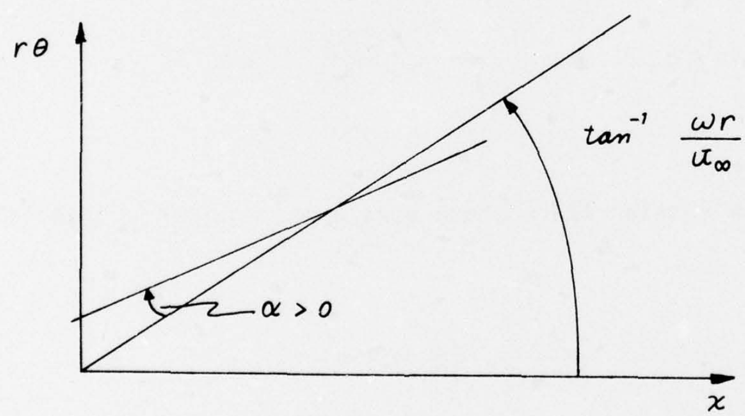
Note that

$$\frac{C_a}{C(r)} = 1 / \sqrt{1 + \rho^2} \qquad (23)$$

In addition, the angle of incidence varies radially as:

$$\alpha = BV(7) + BV(8)\frac{\rho_T}{\rho} + BV(9)\frac{\rho}{\rho_T} \qquad (24)$$

The angle of incidence is measured with respect to the helical direction, as shown below:



For IBC = 2 (the design case) the program version listed here uses a parabolic-arc thickness distribution:

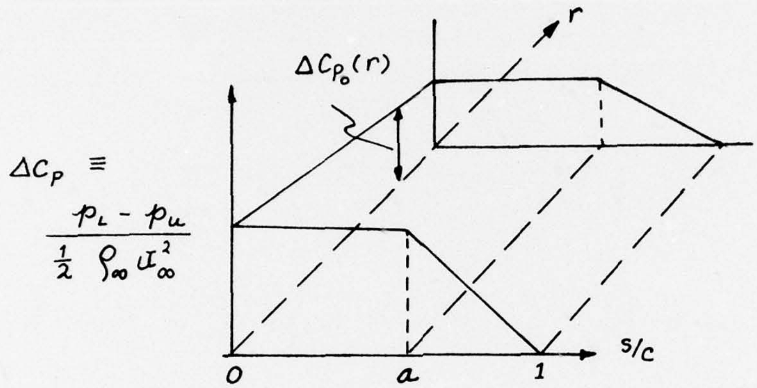
$$t'(s) = 4 \frac{t_{MAX}}{C(r)} \left[1 - 2 \frac{S}{C} \right]$$

$$= 4 \frac{t_{MAX}}{C_a} \frac{1}{\sqrt{1 + \rho^2}} \left[1 - 2 \frac{Z(K)}{\omega C_a / U_\infty} \right] \qquad (25)$$

The maximum thickness is taken as a constant:

$$\frac{t_{MAX}}{c_a} = BV(3) \tag{26}$$

The loading distribution had the form



where the value of $\Delta C_{p_o}(r)$ varies linearly between prescribed values at the hub and tip, and a is constant. These parameters are read in by:

$$BV(2) = a \;, \; BV(1) = \Delta C_{p_o})_{HUB} \;, \; BV(4) = \Delta C_{p_o})_{TIP} \tag{27}$$

As noted in Reference 3, this distribution is inconsistent with the condition $v = 0$ at the hub and tip; the loading $\Delta\phi(r)$ should have zero radial gradient at the hub and tip, if v is to equal zero there.

Other blade shapes and loading distributions can be used, by making appropriate changes in this subroutine, and in the array BV. The sole function of BVAL is to return the arrays DNDS (KB, N, J), J = 1, 2 as defined in Equations (7), (17) and (18) above. Use of other blade-shape or loading distributions does not require changes elsewhere in the program.

Following this calculation of blade-surface boundary conditions, the main iteration loop begins. The outer loop (DO 300 N=...) is in the radial direction. Within this loop, the line on which the solution is being updated is swept downstream in the loop DO 5 K=... . The innermost loop (DO 7 L=...) carries out the relaxation by the standard recursive algorithm, described below.

The finite-difference equation solved in the innermost loop is

$$\left\{V_K^L+\rho^2\right\}(1-\mu_K^L)\left\{f_{K+\frac{1}{2}}\,{}^{N}\phi^{L}_{K+1} - 2f_K\left[\frac{{}^{N}\overset{+}{\phi}{}^{L}_{K}}{\omega_e} + \left(1-\frac{1}{\omega_e}\right){}^{N}\phi^{L}_{K}\right] + f_{K-\frac{1}{2}}\,{}^{N}\overset{+}{\phi}{}^{L}_{K-1}\right\}$$

$$+\,V^L_{K-1}\,\mu^L_{K-1}\left\{f_{K-\frac{1}{2}}\left(2\,{}^{N}\overset{+}{\phi}{}^{L}_{K} - {}^{N}\phi^{L}_{K} - {}^{N}\overset{+}{\phi}{}^{L}_{K-1}\right) - f_{K-\frac{3}{2}}\left({}^{N}\overset{+}{\phi}{}^{L}_{K-1} - {}^{N}\phi^{L}_{K-2}\right)\right\}$$

$$+\,\rho^2\,\mu^L_K\left\{f_{K+\frac{1}{2}}\left({}^{N}\phi^{L}_{K+1} - {}^{N}\phi^{L}_{K}\right) - f_{K-\frac{1}{2}}\left({}^{N}\overset{+}{\phi}{}^{L}_{K} - {}^{N}\overset{+}{\phi}{}^{L}_{K-1}\right)\right\}$$

$$-\,2\,\frac{(1+\rho^2)\,\Delta\tau}{2\,\Delta\zeta}\left\{-{}^{N}\overset{+}{\phi}{}^{L+1}_{K-1} + {}^{N}\phi^{L+1}_{K} + {}^{N}\overset{+}{\phi}{}^{L}_{K-1} - 2\,{}^{N}\phi^{L}_{K} + {}^{N}\phi^{L}_{K+1} + {}^{N}\phi^{L-1}_{K} - {}^{N}\phi^{L-1}_{K+1}\right\}$$

$$+\,\frac{(1+\rho^2)^2}{\rho^2 f_K}\left(\frac{\Delta\tau}{\Delta\zeta}\right)^2\left\{{}^{N}\overset{+}{\phi}{}^{L+1}_{K} - 2\,{}^{N}\overset{+}{\phi}{}^{L}_{K} + {}^{N}\overset{+}{\phi}{}^{L-1}_{K}\right\}$$

$$+\,\frac{1+\rho^2}{f_K}\left(\frac{\Delta\tau}{\Delta\rho}\right)^2\left\{\left(1+\frac{\Delta\rho}{\rho}\right){}^{N+1}\phi^{L}_{K} + \left(1-\frac{\Delta\rho}{\rho}\right){}^{N-1}\phi^{L}_{K} - 2\,{}^{N}\phi^{L}_{K}\right\} = 0 \tag{28}$$

where

$$V^L_K = 1 - M_\infty^2\,(1+\rho^2) - (\gamma+1)\,M_\infty^2\,f_K\,\frac{{}^{N}\phi^{L}_{K+1} - {}^{N}\overset{+}{\phi}{}^{L}_{K-1}}{2\,\Delta\tau}$$

$$\mu^L_K = \begin{matrix}0\\1\end{matrix}\ , \quad \text{for } V^L_K \lessgtr 0 \tag{29}$$

This equation is rewritten as

$$A_K^L \; {}^N\!\overset{+}{\phi}{}_K^{L+1} + B_K^L \; {}^N\!\overset{+}{\phi}{}_K^{L} + C_K^L \; {}^N\!\overset{+}{\phi}{}_K^{L-1} = D_K^L \; , \quad L = 2, 3, \ldots, LMX-1 \tag{30}$$

The solution is found by

$${}^N\!\overset{+}{\phi}{}_K^{L} = EE(L) \; {}^N\!\overset{+}{\phi}{}_K^{L+1} + FF(L) \tag{31}$$

where $EE(L)$ and $FF(L)$ obey the recursion relations

$$EE(L) = -A_K^L \; / \; [B_K^L + C_K^L \, EE(L-1)]$$

$$FF(L) = [D_K^L - C_K^L \, FF(L-1)] \; / \; [B_K^L + C_K^L \, EE(L-1)] \tag{32}$$

Subroutine BVALO uses the blade-surface or periodicity conditions to set EE(1) and FF(1). Then the recursion formulas above are used to find EE(L) and FF(L) for L = 2, 3, ..., LMX - 1. Subroutine BVALI then sets the value of the potential at LMX, by again using the blade-surface or periodicity conditions. Equation (31) is then used to find the values of the potential, from LMX - 1 down to L = 1. These tentative values of ϕ are stored in the array SV; the updated values are found by

$${}^N\!\overset{+}{\phi}{}_K^{L} = \omega \, SV(L) + (1-\omega) \; {}^N\!\phi_K^L \tag{33}$$

The specific formulas used in subroutine BVALO are:

(a) Periodicity condition, for $z < 0$

$$EE(1) = 0 \; , \quad FF(1) = {}^N\!\phi_K^{LMX} \tag{34}$$

(b) Periodicity condition, for $z > \omega c_a / U_\infty$

$$EE(1) = 0 \quad , \quad FF(1) = {}^N\phi_K^{LMX} + \Delta\phi(N) \tag{35}$$

(c) Blade-surface condition for IBC = 1: the derivative $\phi_{\zeta\zeta}$ is approximated by

$$\phi_{\zeta\zeta}\Big)_{L=1} = \frac{6}{\Delta\zeta}\left\{\frac{-\phi_K^3 + 8\phi_K^2 - 7\phi_K^1}{12\,\Delta\zeta} - \frac{1}{2}\,\phi_\zeta\Big)_{L=1}\right\}$$

$$= \frac{6}{\Delta\zeta}\left\{\frac{-\phi_K^3 + 8\phi_K^2 - 7\phi_K^1}{12\,\Delta\zeta} - \frac{\rho}{2}\,\frac{dn}{ds}\Big)_{L=1} - \frac{\rho^2 f_K}{2(1+\rho^2)}\,\frac{2\phi^1_{K+1} + 3\phi^1_K - 6\phi^1_{K-1} + \phi^1_{K-2}}{6\,\Delta\zeta}\right\} \tag{36}$$

This expression is then used to write the potential equation at L = 1 as

$$\left\{V_K^1 + \rho^2\right\}(1-\mu_K^1)\left\{f_{K+\frac{1}{2}}\,{}^N\phi^1_{K+1} - 2f_K\left[\frac{{}^N\overset{+}{\phi}{}^1_K}{\omega_e} + \left(1-\frac{1}{\omega_e}\right){}^N\phi^1_K\right] + f_{K-\frac{1}{2}}\,{}^N\overset{+}{\phi}{}^1_{K-1}\right\}$$

$$+ V^1_{K-1}\,\mu^1_{K-1}\left\{f_{K-\frac{1}{2}}\left(2\,{}^N\overset{+}{\phi}{}^1_K - {}^N\phi^1_K - {}^N\overset{+}{\phi}{}^1_{K-1}\right) - f_{K-\frac{3}{2}}\left({}^N\overset{+}{\phi}{}^1_{K-1} - {}^N\phi^1_{K-2}\right)\right\}$$

$$+ \rho^2\mu^1_K\left\{f_{K+\frac{1}{2}}\left({}^N\phi^1_{K+1} - {}^N\phi^1_K\right) - f_{K-\frac{1}{2}}\left({}^N\overset{+}{\phi}{}^1_K - {}^N\overset{+}{\phi}{}^1_{K-1}\right)\right\}$$

$$- 2(1+\rho^2)\,\frac{\Delta\tau}{4\,\Delta\zeta}\left\{{}^N\phi^3_{K-2} - {}^N\phi^3_K + 8\,{}^N\overset{+}{\phi}{}^2_K - 8\,{}^N\phi^2_{K-1} - \left[4\,{}^N\overset{+}{\phi}{}^1_K + 3\,{}^N\phi^1_K\right] + 8\,{}^N\phi^1_{K-1} - {}^N\phi^1_{K-2}\right\}$$

$$+ \frac{(1+\rho^2)^2}{\rho^2 f_K}\left(\frac{\Delta\tau}{\Delta\zeta}\right)^2\left\{-\frac{1}{2}\,{}^N\phi^3_K + 4\,{}^N\overset{+}{\phi}{}^2_K - \frac{3}{2}\,{}^N\overset{+}{\phi}{}^1_K - 3\rho\,\Delta\zeta\,\frac{dn}{ds}\Big)_{L=1}\right.$$

$$\left. - \frac{\rho^2 f_K}{2(1+\rho^2)}\,\frac{\Delta\zeta}{\Delta\tau}\left[2\,{}^N\phi^1_{K+1} + 3\,{}^N\phi^1_K - 6\,{}^N\phi^1_{K-1} + {}^N\phi^1_{K-2}\right]\right\}$$

$$+ \frac{1+\rho^2}{f_K}\left(\frac{\Delta\tau}{\Delta\rho}\right)^2\left\{\left(1+\frac{\Delta\rho}{\rho}\right){}^{N+1}\phi^1_K + \left(1-\frac{\Delta\rho}{\rho}\right){}^{N-1}\phi^1_K - 2\,{}^N\phi^1_K\right\} = 0 \tag{37}$$

This is then written as

$$AE\ {}^{N}\overset{+}{\phi}{}^{1}_{K} + BE\ {}^{N}\overset{+}{\phi}{}^{2} + CE = 0 \tag{38}$$

which leads to

$$EE(1) = -BE/AE \ , \quad FF(1) = -CE/AE \tag{39}$$

(d) Blade-surface condition for IBC = 2:

$$EE(1) = 0 \ , \quad FF(1) = {}^{N}\phi^{LMX}_{K} + \int_{0}^{z} \frac{\Delta C_{p}}{2}\, dz \tag{40}$$

The latter integral is calculated in BVAL before the iterations begin, and is stored in the array DNDS (K-KLEP, N, 1), where KLEP is the K-value of the station just upstream of the leading edge.

The specific formulas used in subroutine BVALI are:

(a) Periodicity condition, for $z < 0$: points along LMX are treated as field points, with values of the potential on LMX + 1 set equal to their values on L = 2. Thus, Equation (30) is written as

$$B^{LMX}_{K}\ {}^{N}\overset{+}{\phi}{}^{LMX}_{K} + C^{LMX}_{K}\ {}^{N}\overset{+}{\phi}{}^{LMX-1}_{K} = D^{LMX}_{K} - A^{LMX}_{K}\ {}^{N}\phi^{2}_{K} \tag{41}$$

comparison with Eq (31) then leads to the explicit expression:

$$ {}^{N}\overset{+}{\phi}{}^{LMX}_{K} = SV(LMX) = \frac{D^{LMX}_{K} - A^{LMX}_{K}\ {}^{N}\phi^{2}_{K} - C^{LMX}_{K}\ FF(LMX-1)}{B^{LMX}_{K} + C^{LMX}_{K}\ EE(LMX-1)} \tag{42}$$

(b) Periodicity condition, for $z > \omega c_{a}/U_{\infty}$: here the same formulas are used, except that ${}^{N}\phi^{LMX+1}_{K}$ is replaced by ${}^{N}\phi^{2}_{K} - \Delta\phi(N)$, and there is a contribution to $\phi_{\theta\theta}$ from the radial derivative of the circulation:

$$\phi_{\zeta\zeta}^{LMX} = -\frac{\rho^2}{2(1+\rho^2)}\left\{\frac{\Delta\phi(N+1) - 2\Delta\phi(N) + \Delta\phi(N-1)}{(\Delta\rho)^2}\right.$$

$$\left. + \frac{\Delta\phi(N+1) - \Delta\phi(N-1)}{2\rho\Delta\rho}\right\} + \frac{{}^N\phi_K^2 + {}^N\phi_K^{LMX-1} - 2\,{}^N\phi_K^{LMX} - \Delta\phi(N)}{(\Delta\zeta)^2} \tag{43}$$

It should be noted that the evaluation of the mixed derivative at LMX, and at the stations immediately off the blades (K = KLEP and KTEO) uses differences across the blade surface.

(c) Blade-surface condition for IBC = 1: the derivative $\phi_{\zeta\zeta}$ is approximated by

$$\phi_{\zeta\zeta}\big)_{LMX} = \frac{6}{\Delta\zeta}\left\{\frac{-{}^N\phi_K^{LMX-2} + 8\,{}^N\phi_K^{LMX-1} - 7\,{}^N\phi_K^{LMX}}{12\Delta\zeta} + \frac{1}{2}\,\phi_\zeta\big)_{LMX}\right\}$$

$$= \frac{6}{\Delta\zeta}\left\{\frac{-{}^N\phi_K^{LMX-2} + 8\,{}^N\phi_K^{LMX-1} - 7\,{}^N\phi_K^{LMX}}{12\Delta\zeta} + \frac{\rho}{2}\,\frac{dn}{ds}\Big)_{LMX}\right.$$

$$\left. + \frac{\rho^2 f_K}{2(1+\rho^2)}\,\frac{2\,{}^N\phi_{K+1}^{LMX} + 3\,{}^N\phi_K^{LMX} - 6\,{}^N\phi_{K-1}^{LMX} + {}^N\phi_{K-2}^{LMX}}{6\Delta\tau}\right\} \tag{44}$$

Using this expression, the potential equation at LMX is written as:

$$
\begin{aligned}
&\left\{V_K^{LMX}+\rho^2\right\}\left(1-\mu_K^{LMX}\right)\left\{f_{K+\frac{1}{2}}\,{}^{N}\phi_{K+1}^{LMX}-2f_K\left[\frac{{}^{N}\overset{+}{\phi}{}^{LMX}}{\omega_e}+\left(1-\frac{1}{\omega_e}\right){}^{N}\phi_K^{LMX}\right]+f_{K-\frac{1}{2}}\,{}^{N}\overset{+}{\phi}{}_{K-1}^{LMX}\right\} \\
&+V_{K-1}^{LMX}\,\mu_{K-1}^{LMX}\left\{f_{K-\frac{1}{2}}\left(2\,{}^{N}\overset{+}{\phi}{}_K^{LMX}-{}^{N}\phi_K^{LMX}-{}^{N}\overset{+}{\phi}{}_{K-1}^{LMX}\right)-f_{K-\frac{3}{2}}\left({}^{N}\overset{+}{\phi}{}_{K-1}^{LMX}-{}^{N}\phi_{K-2}^{LMX}\right)\right\} \\
&+\rho^2\mu_K^{LMX}\left\{f_{K+\frac{1}{2}}\left({}^{N}\phi_{K+1}^{LMX}-{}^{N}\phi_K^{LMX}\right)-f_{K-\frac{1}{2}}\left({}^{N}\overset{+}{\phi}{}_K^{LMX}-{}^{N}\overset{+}{\phi}{}_{K-1}^{LMX}\right)\right\} \\
&-2(1+\rho^2)\,\frac{\Delta\tau}{4\Delta\zeta}\left\{{}^{N}\phi_{K+2}^{LMX-2}-{}^{N}\phi_K^{LMX-2}+8\,{}^{N}\overset{+}{\phi}{}_K^{LMX-1}-8\,{}^{N}\phi_{K+1}^{LMX-1}\right. \\
&\qquad\qquad\left.+4\,{}^{N}\phi_{K+1}^{LMX}-{}^{N}\overset{+}{\phi}{}_K^{LMX}-4\,{}^{N}\phi_{K-1}^{LMX}+{}^{N}\phi_{K-2}^{LMX}\right\} \\
&+\frac{(1+\rho^2)^2}{\rho^2 f_K}\left(\frac{\Delta\tau}{\Delta\zeta}\right)^2\left\{-\frac{1}{2}\,{}^{N}\phi_K^{LMX-2}+4\,{}^{N}\overset{+}{\phi}{}_K^{LMX-1}-\frac{7}{2}\,{}^{N}\overset{+}{\phi}{}_K^{LMX}+3\rho\Delta\zeta\,\frac{d\eta}{ds}\Big)_{LMX}\right. \\
&\qquad\left.+\frac{\rho^2 f_K}{2(1+\rho^2)}\,\frac{\Delta\zeta}{\Delta\tau}\left[2\,{}^{N}\phi_{K+1}^{LMX}+3\,{}^{N}\phi_K^{LMX}-6\,{}^{N}\phi_{K-1}^{LMX}+{}^{N}\phi_{K-2}^{LMX}\right]\right\} \\
&+\frac{1+\rho^2}{f_K}\left(\frac{\Delta\tau}{\Delta\rho}\right)^2\left\{\left(1+\frac{\Delta\rho}{\rho}\right){}^{N+1}\phi_K^{LMX}+\left(1-\frac{\Delta\rho}{\rho}\right){}^{N-1}\phi_K^{LMX}-2\,{}^{N}\phi_K^{LMX}\right\}=0
\end{aligned}
\tag{45}
$$

This is written in the form

$$
AE\,{}^{N}\overset{+}{\phi}{}_K^{LMX}+BE\,{}^{N}\overset{+}{\phi}{}_K^{LMX-1}+CE=0 \tag{46}
$$

and is solved, along with Equation (31), to give

$$ {}^{N}\overset{+}{\phi}{}_{K}^{LMX} = SV(LMX) = -\frac{CE + BE \cdot FF(LMX-1)}{AE + BE \cdot EE(LMX-1)} \tag{47} $$

(d) Blade-surface condition for IBC = 2: here the derivative $\phi_{\zeta\zeta}$ is approximated by using

$$ \phi_{\zeta}\Big)_{LMX} = \phi_{\zeta}\Big)_{L=1} - \frac{\rho^2}{1+\rho^2}\,\frac{\Delta C_p}{2} - \rho t'(s) \tag{48} $$

This equation, substituted into the first form of Equation (44), leads to the following form of the potential equation

$$ \begin{aligned} &\left\{V_K^{LMX} + \rho^2\right\}(1-\mu_K^{LMX})\left\{f_{K+\frac{1}{2}}\,{}^{N}\phi_{K+1}^{LMX} - 2f_K\left[\frac{{}^{N}\overset{+}{\phi}{}_K^{LMX}}{\omega_e} + \left(1-\frac{1}{\omega_e}\right){}^{N}\phi_K^{LMX}\right] + f_{K-\frac{1}{2}}\,{}^{N}\overset{+}{\phi}{}_{K-1}^{LMX}\right\} \\ &+ V_{K-1}^{LMX}\,\mu_{K-1}^{LMX}\left\{f_{K-\frac{1}{2}}\left(2\,{}^{N}\overset{+}{\phi}{}_K^{LMX} - {}^{N}\phi_K^{LMX} - {}^{N}\overset{+}{\phi}{}_{K-1}^{LMX}\right) - f_{K-\frac{3}{2}}\left({}^{N}\overset{+}{\phi}{}_{K-1}^{LMX} - {}^{N}\phi_{K-2}^{LMX}\right)\right\} \\ &+ \rho^2\mu_K^{LMX}\left\{f_{K+\frac{1}{2}}\left({}^{N}\phi_{K+1}^{LMX} - {}^{N}\phi_K^{LMX}\right) - f_{K-\frac{1}{2}}\left({}^{N}\overset{+}{\phi}{}_K^{LMX} - {}^{N}\overset{+}{\phi}{}_{K-1}^{LMX}\right)\right\} \\ &-2(1+\rho^2)\,\frac{\Delta\tau}{4\Delta\zeta}\left\{{}^{N}\phi_{K+2}^{LMX-2} - {}^{N}\phi_K^{LMX-2} + 8\,{}^{N}\overset{+}{\phi}{}_K^{LMX-1} - 8\,{}^{N}\phi_{K+1}^{LMX-1}\right. \\ &\qquad\qquad \left. + 4\,{}^{N}\phi_{K+1}^{LMX} - {}^{N}\overset{+}{\phi}{}_K^{LMX} - 4\,{}^{N}\phi_{K-1}^{LMX} + {}^{N}\phi_{K-2}^{LMX}\right\} \\ &+ \frac{(1+\rho^2)^2}{\rho^2 f_K}\left(\frac{\Delta\tau}{\Delta\zeta}\right)^2\left\{-\frac{1}{2}\,{}^{N}\phi_K^{LMX-2} + 4\,{}^{N}\overset{+}{\phi}{}^{LMX-1} - \frac{7}{2}\,{}^{N}\overset{+}{\phi}{}_K^{LMX}\right. \\ &\qquad\qquad \left. + \frac{1}{2}\left[-6\Delta\zeta\left\langle -\frac{\rho^2}{1+\rho^2}\,\frac{\Delta C_p}{2} - \rho t'(s)\right\rangle - 11\,{}^{N}\phi_K^{1} + 18\,{}^{N}\phi_K^{2} - 9\,{}^{N}\phi_K^{3} + 2\,{}^{N}\phi_K^{4}\right]\right\} \\ &+ \frac{1+\rho^2}{f_K}\left(\frac{\Delta\tau}{\Delta\rho}\right)^2\left\{\left(1+\frac{\Delta\rho}{\rho}\right){}^{N+1}\phi_K^{LMX} + \left(1-\frac{\Delta\rho}{\rho}\right){}^{N-1}\phi_K^{LMX} - 2\,{}^{N}\phi_K^{LMX}\right\} = 0 \end{aligned} \tag{49} $$

The loading and thickness values are calculated in BVAL, and are stored in:

$$DNDS(K-KLEP, N, 2) = \frac{\rho^2}{1+\rho^2}\frac{\Delta C_p}{2} + \rho t'(s) \tag{50}$$

This line relaxation procedure is carried out up to KMX - 1. Then values of the potential at KMX are set equal to the value required by mass conservation:

$$\overset{N+L}{\phi}_{KMX} = \frac{C - DAF \cdot {}^{N}\overset{+}{\phi}{}^{L}_{KMX-2} - DBF \cdot {}^{N}\overset{+}{\phi}{}^{L}_{KMX-1}}{DCF} \tag{51}$$

where the following coefficients are calculated in GRID

$$DAF = (z_{KMX} - z_{KMX-1})/(z_{KMX-1} - z_{KMX-2})(z_{KMX} - z_{KMX-2})$$

$$DBF = (z_{KMX} - z_{KMX-2})/(z_{KMX-1} - z_{KMX-2})(z_{KMX-1} - z_{KMX})$$

$$DCF = (2z_{KMX} - z_{KMX-1} - z_{KMX-2})/(z_{KMX} - z_{KMX-2})(z_{KMX} - z_{KMX-1}) \tag{52}$$

Next, the circulation is updated, using

$$\overset{+}{\Delta}\phi(N) = OBV\,\Delta\phi^* + (1 - OBV)\,\Delta\phi(N) \tag{53}$$

where

$$\Delta\phi^* = \frac{\left\{-(z_{TE} - z_b)^2\Delta\phi_a + (z_{TE} - z_a)^2\Delta\phi_b\right\}}{(z_b - z_a)(2z_{TE} - z_a - z_b)}$$

$$z_{TE} = \omega c_a / u_\infty$$

$$\Delta\phi_{a,b} = \overset{+}{\phi}(z_{a,b}, \rho, 0) - \overset{+}{\phi}\left(z_{a,b}, \rho, \frac{2\pi}{\beta}\right) \tag{54}$$

This completes a sweep in the z- direction. The iteration counter ITK is then incremented, and compared with ITKMX, the maximum number of z - sweeps for each radial one. After ITKMX sweeps are done, the solution moves to the next radial station, where the process is repeated for N = 2, 3, ... NMX - 1. The iteration counter ITR is used to number the radial sweeps; a total of ITRMX is allowed.

After each radial sweep, values of ϕ and $\Delta\phi$ are updated at N = 1 and N = NMX according to

$$
\begin{aligned}
{}^{1}\phi_K^{L} &= \frac{4}{3}\,{}^{2}\phi_K^{L} - \frac{1}{3}\,{}^{3}\phi_K^{L} \\
{}^{NMX}\phi_K^{L} &= \frac{4}{3}\,{}^{NMX-1}\phi_K^{L} - \frac{1}{3}\,{}^{NMX-2}\phi_K^{L}
\end{aligned}
\tag{55}
$$

$$
\begin{aligned}
\Delta\phi(1) &= 2\,\Delta\phi(2) - \Delta\phi(3) \\
\Delta\phi(NMX) &= 2\,\Delta\phi(NMX-1) - \Delta\phi(NMX-2)
\end{aligned}
\tag{56}
$$

Also updated at the end of each radial sweep is the quantity C, used [see Equation (10)] in enforcing mass conservation at the downstream edge of the grid. This quantity, called CONST, is evaluated by the trapezoidal rule.

The OUTPUT subroutine can be called at intervals of JPRT in the ITK iterations and at intervals of NPRT in the ITR iterations. The former choice is intended for diagnostic purposes. It would normally be used only for checkout purposes, and even then for very few iterations and very few radial stations, since it generates many lines of output. To avoid this diagnostic output, set JPRT > ITKMX.

Several options are available in the output; these are controlled by the indicator IOP, and are described in Appendix A. The usual sequence is to run the solution for a certain number of iterations, write the solution on tape (ISAVE = 1), examine the output, and then restart the solution (ISTART = 1) with adjusted values of the relaxation factors, iteration-count parameters, etc. Some of the output options allow a minimum number of lines to be printed during these intermediate stages.

One special option is IOP = 4, which calculates Mach number contours. This option can only be used with a solution found on a previous run: no iterations are made, and the Mach number calculation is done on the values read in from the tape.

The quantities calculated and displayed by subroutine OUTPUT include the perturbation velocities u_s/W_o , u_n/W_o , v/U_∞ and the local Mach number. These are defined as follows:

$$\frac{u_s}{W_o} = \frac{\partial\phi/\partial z}{1+\rho^2} = f_K \frac{({}^N\phi^L_{K+1} - {}^N\phi^L_{K-1})}{(1+\rho^2)\,2\Delta\tau} \tag{57}$$

$$\frac{v}{U_\infty} = \frac{{}^{N+1}\phi^L_K - {}^{N-1}\phi^L_K}{2\Delta\rho} \tag{58}$$

$$\frac{u_n}{W_o} = \frac{1}{\rho}\frac{\partial\phi}{\partial\zeta} - \frac{\rho}{1+\rho^2}\frac{\partial\phi}{\partial z}$$

$$= \frac{{}^N\phi^{L+1}_{K-1} - {}^N\phi^L_{K-1} + {}^N\phi^L_{K+1} - {}^N\phi^{L-1}_{K+1}}{2\rho\Delta\zeta} - \rho\frac{u_s}{W_o} \tag{59}$$

The first term in this expression is equal to $\partial\phi/\partial\zeta$, with a truncation error $O(\Delta\zeta)^2$. This particular form is shown so that:

$$\frac{u_n}{W_o} = \frac{{}^N\phi_{K-1}^{L+1} - {}^N\phi_{K+1}^{L-1}}{2\rho\Delta\zeta} = \frac{{}^N\phi_{K-1}^{L+1} - {}^N\phi_{K+1}^{L-1}}{2\sqrt{1+\rho^2}\,\Delta\hat{n}} \tag{60}$$

when

$$\frac{\rho\Delta\zeta}{\Delta z} = \frac{1+\rho^2}{\rho} \tag{61}$$

An alternate expression for $\partial\phi/\partial\zeta$, having the same truncation error, was used in References 1-4, namely:

$$\frac{1}{\rho}\frac{\partial\phi}{\partial\zeta} = \frac{\phi_K^{L+1} - \phi_K^{L-1}}{2\rho\Delta\zeta} \tag{62}$$

This formula tends to give erratic results when shock waves are present, since it maximizes the amount of differencing done across the shock.

The local Mach number is defined by equating the coefficient of ϕ_{ss} in Equation (4) to $1-M^2$:

$$1 - M^2 = 1 - M_\infty^2(1+\rho^2) - (\gamma+1)M_\infty^2\phi_z$$

or

$$M^2 = M_{rel}^2\left[1 + (\gamma+1)\,u_s/W_o\right]$$

Thus

$$M = M_{rel}\sqrt{1+(\gamma+1)\,u_s/W_o} \approx M_{rel}\left[1 + \frac{\gamma+1}{2}\frac{u_s}{W_o}\right] \tag{63}$$

The square-root formula was used in References 1-4; the present report uses the linearized version.

At the very end of every calculation, the output subroutine is called with IOP = 5. This displays the following performance data at each radius:

the values of N, $\rho(N)$, $\Delta\phi(N)$

$$w/v_\infty)_{z\to\infty} = \frac{-B}{2\pi\rho}\,\Delta\phi \tag{64}$$

$$u/v_\infty)_{z\to\infty} = C - \rho\,\frac{w}{v_\infty}\Big)_{z\to\infty} \tag{65}$$

the turning angle, defined as $\tan^{-1}\dfrac{\left[\dfrac{w}{v_\infty} - \rho\,\dfrac{u}{v_\infty}\right]_{z\to\infty}}{1+\rho^2}$ (66)

the total pressure ratio $\dfrac{p_{02}}{p_{01}} = 1 + \dfrac{\gamma M_\infty^2}{1+\dfrac{\gamma-1}{2}M_\infty^2}\,\dfrac{B}{2\pi}\,\Delta\phi$ (67)

The convergence of the solution can be monitored by calculating the residuals. This is done, at ITR intervals of IRXP, in subroutine RESID. This subroutine evaluates all five terms in the potential equation at the interior points of the grid, i.e., for N = 2, 3, ..., NMX - 1, K = 2, 3, ..., KMX - 1, and L = 2, 3, ..., LMX - 1. The five terms (called A through E) are evaluated as follows [compare with Equation (4)]:

$$A = (\Delta\tau)^2\left\{1 - M_\infty^2(1+\rho^2) - (\gamma+1)M_\infty^2\,\phi_z\right\}\phi_{\tau\tau} \tag{68}$$

$$\text{B} = \rho^2 (\Delta\tau)^2 \phi_{\tau\tau} \tag{69}$$

$$\text{C} = -2(1+\rho^2)(\Delta\tau)^2 \phi_{\xi\tau} \tag{70}$$

$$\text{D} = \frac{(1+\rho^2)^2}{\rho^2 f}(\Delta\tau)^2 \phi_{\xi\xi} \tag{71}$$

$$\text{E} = \frac{1+\rho^2}{\rho f}\frac{\partial}{\partial\rho}(\rho\phi_\rho) \tag{72}$$

The residual at each point is defined as

$$^{N}R_K^L = \left| \text{A} + \text{B} + \text{C} + \text{D} + \text{E} \right| \tag{73}$$

At each value of N, the value of the maximum residual is printed, along with the K and L values of the point where it occurs. Also shown is the average residual, i.e., the mean of $^{N}R_K^L$ over the range N = 2, NMX - 1, K = 2, KMX - 1; L = 2, LMX - 1. The magnitudes of the residuals themselves are only meaningful in relation to the sizes of the individual terms whose sum they represent. Thus the mean value of ϕ and of the sum of the absolute values of the five terms are also shown, i.e.:

$$AVG\ PHI \equiv \frac{1}{P} \sum_{\substack{N=2,\ NMX-1 \\ K=2,\ KMX-1 \\ L=2,\ LMX-1}} \left| ^{N}\phi_K^L \right| \tag{74}$$

$$AVG\ SUM \equiv \frac{1}{P} \sum_{\substack{N=2,\,NMX-1\\ K=2,\,KMX-1\\ L=2,\,LMX-1}} \left\{ |A| + |B| + |C| + |D| + |E| \right\} \qquad (75)$$

where $$P = (NMX - 2)\ (KMX - 2)\ (LMX - 2) \qquad (76)$$

A second way to monitor the convergence of the solution is to observe the perturbation velocities at a number of selected points. Our experience has been that these will usually converge to three decimal places when AVG RES/AVG SUM $\approx 3 \times 10^{-2}$.

If IRXP is prefixed by a minus sign, subroutine RESID will display the values of all five terms and the residual, at each of the interior grid points. This option, which produces many lines of output, can be used for diagnostic purposes.

If IBC = 2, subroutine OUTPUT also calculates the blade shape, using the formula

$$\frac{n_{u,L}}{C_a} = \frac{1+\rho^2}{\omega C_a / U_\infty} \int_0^z \left.\frac{u_n}{W_0}\right)_{u,L} dz \qquad (77)$$

The US array is used to store this integral, which is done by the trapezoidal rule. The quantities printed are

$$z,\ \frac{z}{\frac{\omega C_a}{U_\infty}},\ \frac{n_u}{C_a},\ \frac{n_L}{C_a},\ \frac{h}{C_a},\ \frac{t}{C_a},\ \frac{n_u}{C(r)},\ \frac{n_L}{C(r)},\ \frac{h}{C(r)},\ \frac{t}{C(r)}$$

Section 3
SAMPLE CASE

To illustrate the operation of the program, some details are presented of the two-dimensional case shown in Figure 7 of Reference 4. This cascade had a stagger angle of 45°, an inlet relative Mach number of 0.9, a solidity C_a/L_T of 0.411, six percent thickness-to-chord ratio, and no camber or thickness. The input data were as follows:

$B = 10$, $h = 0.9$, $C_a/L_T = 0.41073$, $M_x = 0.636396$, $M_{\theta_{tip}} = 0.669891$

$\gamma = 1.4$, $FXB = -2$, $FXI = +2$, $RXE = 1$, $RXH = 0.9$

$OBV = 0.05$

$KMX = 60$, $LMX = 30$, $NMX = 3$, $IBC = 1$, $IDM = 2$, $ITKMX = 360$,

$JPRT = 400$, $ISTART = 0$, $ISAVE = 0$, $IOP = 1$, $ITRMX = 1$,

$NPRT = 0$, $ITPR = 1$, $IRXP = 1$

$BV(1) = 0.0848528$, $BV(2)$ through $BV(10) = 0.$

Figures 3a - 3c are the first three pages of the output, showing the run conditions, coordinate system, and blade geometry. Figures 4a - 4d are portions of the pages on which the potential, streamwise, normal and radial velocity components appear. The last of these is zero for the case shown; it would be non-zero if NMX were greater than 3, i.e., if two-dimensional strip-theory calculations were being done at a series of radial stations. Figure 5 is

the last page of the output, showing the most recent value of the circulation, the residuals, and the performance data. Note that the average residual is already down to less than 1/100 of the average sum of terms, even at this early stage of the iterations.

This case used 3.5 minutes of CPU time on an IBM 360/65.

REFERENCES

1. Rae, W.J. "Nonlinear Small-Disturbance Equations for Three-Dimensional Transonic Flow Through a Compressor Blade Row" AFOSR TR-76-1082 AD-A031234 (August 1976).

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3. Rae, W.J. "Finite-Difference Calculations of Three-Dimensional Transonic Flow Through a Compressor Blade Row, Using the Small-Disturbance Nonlinear Potential Equation" pp. 228-252 of <u>Transonic Flow Problems in Turbomachinery</u> ed. by T.C. Adamson & M.F. Platzer Hemisphere Publishing Corp., Washington (1977).

4. Rae, W.J. "Calculations of Three-Dimensional Transonic Compressor Flowfields by a Relaxation Method" Journal of Energy 1 (1977) 284-296.

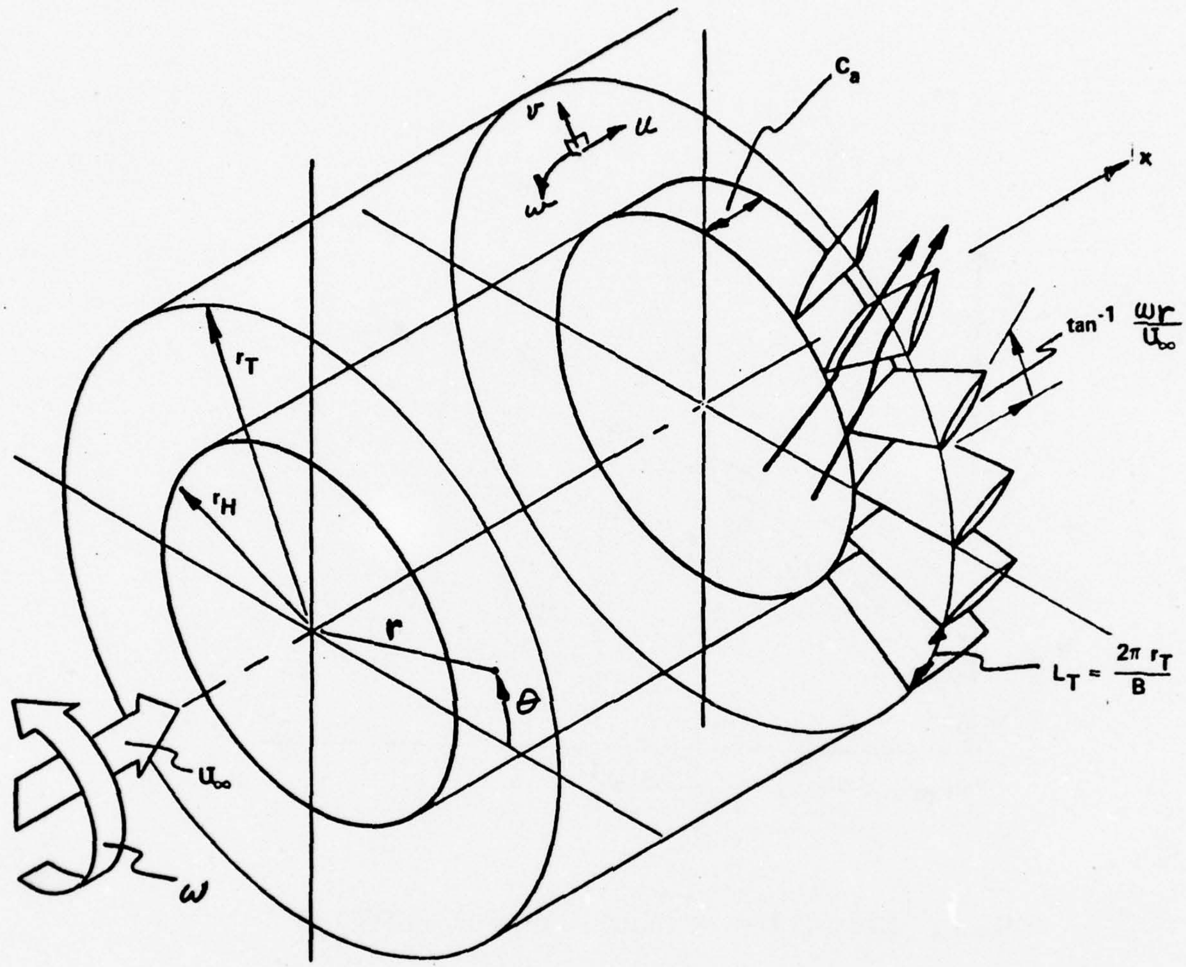

Figure 1 **BLADE-FIXED COORDINATES**

ROTOR IS STATIONARY IN A HELICAL APPROACH FLOW

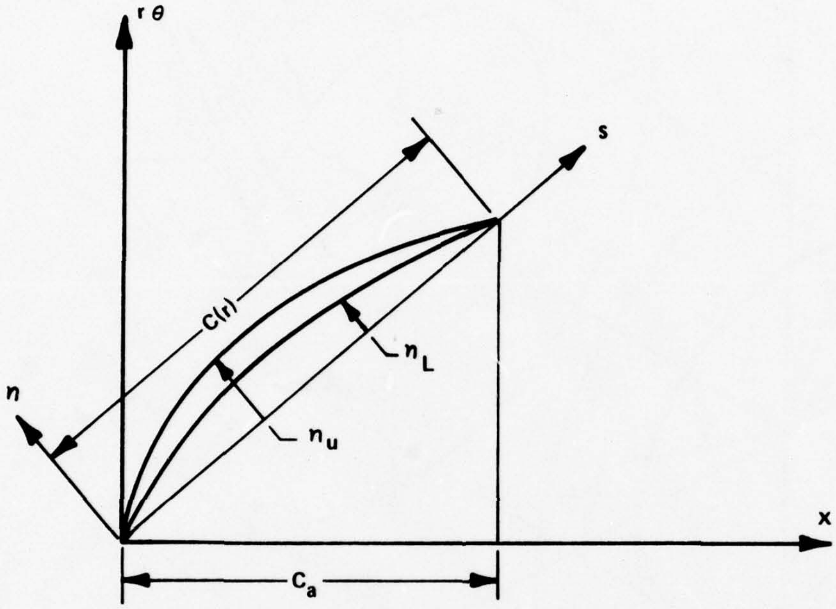

Figure 2 DEFINITIONS OF BLADE-SURFACE GEOMETRY

SAMPLE CASE FOR USERS GUIDE REPORT
CASE IS THAT OF FIGURE 7, JOUNAL OF ENERGY, P. 289, SEPT/OCT 1977

THIS BLADE ROW HAS 10 BLADES, WITH A HUB-TO-TIP RATIO OF 0.900 AND SOLIDITY CA/LT = 0.411

AXIAL MACH NO. = 0.636, TANGENTIAL MACH NO AT THE TIP = 0.670, TOTAL MACH NO. AT THE TIP = 0.924

SPECIFIC HEAT RATIO = 1.400

THIS IS A 2-DIMENSIONAL CALCULATION, WITH GRID SIZE KMX/LMX/NMX = 60/ 30/ 3

RELAXATION FACTORS FOR ELLIPTIC AND HYPERBOLIC POINTS ARE LISTED BELOW AS RXE AND RXH

THE BLADES LIE BETWEEN Z = 0 AND 0.271652

K	Z	X/CA	RXE	RXH
1	-0.543304	-2.0000	1.0000	0.9000
2	-0.425544	-1.5665	1.0000	0.9000
3	-0.355454	-1.3085	1.0000	0.9000
4	-0.304851	-1.1222	1.0000	0.9000
5	-0.264902	-0.9752	1.0000	0.9000
6	-0.231672	-0.8528	1.0000	0.9000
7	-0.203059	-0.7475	1.0000	0.9000
8	-0.177810	-0.6546	1.0000	0.9000
9	-0.155114	-0.5710	1.0000	0.9000
10	-0.134416	-0.4948	1.0000	0.9000
11	-0.115323	-0.4245	1.0000	0.9000
12	-0.097539	-0.3591	1.0000	0.9000
13	-0.080842	-0.2976	1.0000	0.9000
14	-0.065058	-0.2395	1.0000	0.9000
15	-0.050047	-0.1842	1.0000	0.9000
16	-0.035696	-0.1314	1.0000	0.9000
17	-0.021913	-0.0807	1.0000	0.9000
18	-0.008619	-0.0317	1.0000	0.9000
19	0.004253	0.0157	1.0000	0.9000
20	0.016758	0.0617	1.0000	0.9000
21	0.028946	0.1066	1.0000	0.9000
22	0.040862	0.1504	1.0000	0.9000
23	0.052544	0.1934	1.0000	0.9000
24	0.064027	0.2357	1.0000	0.9000
25	0.075343	0.2773	1.0000	0.9000
26	0.086521	0.3185	1.0000	0.9000
27	0.097589	0.3592	1.0000	0.9000
28	0.108573	0.3997	1.0000	0.9000
29	0.119498	0.4399	1.0000	0.9000
30	0.130387	0.4800	1.0000	0.9000

FIGURE 3a

SAMPLE CASE FOR USERS GUIDE REPORT
CASE IS THAT OF FIGURE 7, JOUNAL OF ENERGY, P. 289, SEPT/OCT 1977 (Cont.)

K	Z	X/CA	RXE	RXH
31	0.141264	0.5200	1.0000	0.9000
32	0.152154	0.5601	1.0000	0.9000
33	0.163079	0.6003	1.0000	0.9000
34	0.174063	0.6408	1.0000	0.9000
35	0.185131	0.6815	1.0000	0.9000
36	0.196309	0.7226	1.0000	0.9000
37	0.207625	0.7643	1.0000	0.9000
38	0.219108	0.8066	1.0000	0.9000
39	0.230790	0.8496	1.0000	0.9000
40	0.242705	0.8934	1.0000	0.9000
41	0.254894	0.9383	1.0000	0.9000
42	0.267399	0.9843	1.0000	0.9000
43	0.280270	1.0317	1.0000	0.9000
44	0.293564	1.0807	1.0000	0.9000
45	0.307348	1.1314	1.0000	0.9000
46	0.321698	1.1842	1.0000	0.9000
47	0.336709	1.2395	1.0000	0.9000
48	0.352494	1.2976	1.0000	0.9000
49	0.369191	1.3591	1.0000	0.9000
50	0.386974	1.4245	1.0000	0.9000
51	0.406068	1.4948	1.0000	0.9000
52	0.426766	1.5710	1.0000	0.9000
53	0.449461	1.6545	1.0000	0.9000
54	0.474711	1.7475	1.0000	0.9000
55	0.503323	1.8528	1.0000	0.9000
56	0.536554	1.9752	1.0000	0.9000
57	0.576502	2.1222	1.0000	0.9000
58	0.627105	2.3085	1.0000	0.9000
59	0.697193	2.5665	1.0000	0.9000
60	0.814954	3.0000	1.0000	0.9000

FIGURE 3a (Cont.)

L	ZETA
1	0.0
2	0.0217
3	0.0433
4	0.0650
5	0.0867
6	0.1083
7	0.1300
8	0.1517
9	0.1733
10	0.1950
11	0.2167
12	0.2383
13	0.2600
14	0.2817
15	0.3033
16	0.3250
17	0.3467
18	0.3683
19	0.3900
20	0.4117
21	0.4333
22	0.4550
23	0.4767
24	0.4983
25	0.5200
26	0.5417
27	0.5633
28	0.5850
29	0.6067
30	0.6283

OPTIMUM SHOCK CAPTURING OCCURS WHEN THE GRID-SIZE RATIO RHO*(DELTA ZETA)/(DELTA Z) = (1+RHO**2)/RHO
THE TABLE BELOW LISTS THESE OPTIMUM VALUES, COMPARED TO THE VALUES ACTUALLY USED AT K =30

N	RHO	US/WO CRIT	ARCTAN(RHO), DEGREES	M REL	GRID-SIZE RATIO OPTIMUM	GRID-SIZE RATIO ACTUAL(AT K = 30)
1	0.9474	1.2552E-01	43.452	0.8766	2.0029	1.8871
2	1.0000	9.7737E-02	45.000	0.9000	2.0000	1.9920
3	1.0526	7.1374E-02	46.469	0.9240	2.0026	2.0968

FIGURE 3b

```
BLADE GEOMETRY AND ANGLE OF ATTACK SPECIFIED ARE
               T MAX/CAX = TA + TB/R + TC*R, R = RO(N)/RTIP
                    WHERE TA =     8.485E-02 TB =     0.0        TC =     0.0

               H MAX/CAX = HA + HB/R + HC*R
                    WHERE HA =     0.0       HB =     0.0        HC =     0.0

               ALPHA = AA + AB/R + AC*R    (DEGREES)
                    WHERE AA =     0.0       AB =     0.0        AC =     0.0
```

FIGURE 3c

VALUES OF THE POTENTIAL AFTER ITR = 1, ITK = 360 AT RHO(2) = 1.0000

K	L= 1	2	3	4	5	6	7	8	9	10
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	-1.470E-03	-1.503E-03	-1.529E-03	-1.547E-03	-1.556E-03	-1.557E-03	-1.548E-03	-1.531E-03	-1.506E-03	-1.474E-03
3	-2.653E-03	-2.649E-03	-2.624E-03	-2.579E-03	-2.518E-03	-2.441E-03	-2.354E-03	-2.260E-03	-2.163E-03	-2.068E-03
4	-3.414E-03	-3.299E-03	-3.163E-03	-3.010E-03	-2.849E-03	-2.687E-03	-2.531E-03	-2.390E-03	-2.269E-03	-2.175E-03
5	-3.630E-03	-3.396E-03	-3.159E-03	-2.930E-03	-2.719E-03	-2.538E-03	-2.395E-03	-2.296E-03	-2.246E-03	-2.248E-03
6	-3.389E-03	-3.090E-03	-2.822E-03	-2.600E-03	-2.433E-03	-2.328E-03	-2.291E-03	-2.323E-03	-2.423E-03	-2.585E-03
7	-2.896E-03	-2.621E-03	-2.418E-03	-2.298E-03	-2.263E-03	-2.314E-03	-2.450E-03	-2.664E-03	-2.947E-03	-3.290E-03
8	-2.379E-03	-2.222E-03	-2.169E-03	-2.222E-03	-2.377E-03	-2.626E-03	-2.961E-03	-3.370E-03	-3.838E-03	-4.348E-03
9	-2.028E-03	-2.057E-03	-2.296E-03	-2.469E-03	-2.834E-03	-3.292E-03	-3.828E-03	-4.424E-03	-5.056E-03	-5.694E-03
10	-1.954E-03	-2.202E-03	-2.573E-03	-3.055E-03	-3.636E-03	-4.301E-03	-5.028E-03	-5.789E-03	-6.543E-03	-7.240E-03
11	-2.195E-03	-2.670E-03	-3.266E-03	-3.970E-03	-4.767E-03	-5.634E-03	-6.534E-03	-7.415E-03	-8.209E-03	-8.850E-03
12	-2.744E-03	-3.450E-03	-4.276E-03	-5.209E-03	-6.225E-03	-7.282E-03	-8.309E-03	-9.213E-03	-9.905E-03	-1.034E-02
13	-3.591E-03	-4.536E-03	-5.608E-03	-6.785E-03	-8.021E-03	-9.226E-03	-1.026E-02	-1.101E-02	-1.142E-02	-1.151E-02
14	-4.731E-03	-5.941E-03	-7.291E-03	-8.730E-03	-1.016E-02	-1.137E-02	-1.218E-02	-1.256E-02	-1.256E-02	-1.224E-02
15	-6.187E-03	-7.713E-03	-9.397E-03	-1.111E-02	-1.257E-02	-1.344E-02	-1.377E-02	-1.366E-02	-1.319E-02	-1.244E-02
16	-8.014E-03	-9.958E-03	-1.205E-02	-1.387E-02	-1.482E-02	-1.507E-02	-1.482E-02	-1.419E-02	-1.327E-02	-1.214E-02
17	-1.034E-02	-1.292E-02	-1.537E-02	-1.635E-02	-1.643E-02	-1.605E-02	-1.524E-02	-1.414E-02	-1.282E-02	-1.136E-02
18	-1.347E-02	-1.719E-02	-1.809E-02	-1.802E-02	-1.738E-02	-1.635E-02	-1.505E-02	-1.355E-02	-1.191E-02	-1.016E-02
19	-1.977E-02	-2.012E-02	-1.971E-02	-1.880E-02	-1.754E-02	-1.602E-02	-1.432E-02	-1.249E-02	-1.058E-02	-8.617E-03
20	-2.246E-02	-2.157E-02	-2.032E-02	-1.879E-02	-1.704E-02	-1.514E-02	-1.312E-02	-1.102E-02	-8.899E-03	-6.788E-03
21	-2.359E-02	-2.196E-02	-2.013E-02	-1.813E-02	-1.600E-02	-1.379E-02	-1.151E-02	-9.216E-03	-6.948E-03	-4.721E-03
22	-2.369E-02	-2.154E-02	-1.927E-02	-1.691E-02	-1.451E-02	-1.203E-02	-9.570E-03	-7.141E-03	-4.757E-03	-2.444E-03
23	-2.303E-02	-2.048E-02	-1.788E-02	-1.528E-02	-1.260E-02	-9.962E-03	-7.370E-03	-4.812E-03	-2.333E-03	2.313E-05
24	-2.174E-02	-1.889E-02	-1.609E-02	-1.320E-02	-1.039E-02	-7.616E-03	-4.887E-03	-2.241E-03	2.894E-04	2.550E-03
25	-1.995E-02	-1.694E-02	-1.384E-02	-1.084E-02	-7.882E-03	-4.982E-03	-2.167E-03	5.376E-04	3.016E-03	4.646E-03
26	-1.781E-02	-1.450E-02	-1.130E-02	-8.164E-03	-5.091E-03	-2.106E-03	7.713E-04	3.464E-03	5.383E-03	5.931E-03
27	-1.517E-02	-1.178E-02	-8.457E-03	-5.209E-03	-2.053E-03	9.959E-04	3.896E-03	6.141E-03	6.752E-03	6.779E-03
28	-1.226E-02	-8.753E-03	-5.329E-03	-2.000E-03	1.219E-03	4.316E-03	6.893E-03	7.632E-03	7.652E-03	7.299E-03
29	-9.044E-03	-5.442E-03	-1.940E-03	1.449E-03	4.732E-03	7.624E-03	8.585E-03	8.578E-03	8.222E-03	7.534E-03
30	-5.536E-03	-1.862E-03	1.697E-03	5.158E-03	8.338E-03	9.612E-03	9.556E-03	9.200E-03	8.499E-03	7.495E-03
31	-1.754E-03	1.974E-03	5.607E-03	9.044E-03	1.070E-02	1.058E-02	1.024E-02	9.529E-03	8.487E-03	7.196E-03
32	2.294E-03	6.095E-03	9.756E-03	1.182E-02	1.166E-02	1.133E-02	1.063E-02	9.548E-03	8.188E-03	6.656E-03
33	6.640E-03	1.049E-02	1.295E-02	1.277E-02	1.249E-02	1.179E-02	1.068E-02	9.250E-03	7.618E-03	5.911E-03
34	1.127E-02	1.404E-02	1.392E-02	1.370E-02	1.304E-02	1.190E-02	1.039E-02	8.642E-03	6.812E-03	5.007E-03
35	1.507E-02	1.510E-02	1.497E-02	1.436E-02	1.321E-02	1.162E-02	9.735E-03	7.757E-03	5.824E-03	4.004E-03
36	1.630E-02	1.630E-02	1.577E-02	1.463E-02	1.294E-02	1.091E-02	8.750E-03	6.658E-03	4.724E-03	2.961E-03
37	1.770E-02	1.727E-02	1.615E-02	1.438E-02	1.217E-02	9.796E-03	7.506E-03	5.434E-03	3.585E-03	1.930E-03
38	1.887E-02	1.779E-02	1.596E-02	1.355E-02	1.090E-02	8.350E-03	6.124E-03	4.177E-03	2.470E-03	9.563E-04
39	1.956E-02	1.769E-02	1.507E-02	1.207E-02	9.205E-03	6.775E-03	4.723E-03	2.961E-03	1.427E-03	7.679E-05
40	1.959E-02	1.677E-02	1.334E-02	1.001E-02	7.360E-03	5.201E-03	3.391E-03	1.840E-03	4.932E-04	-6.785E-04

FIGURE 4a

VALUES OF THE POTENTIAL AFTER ITR = 1, ITK = 360 AT RHO(2) = 1.0000 (Cont.)

K	L= 1	2	3	4	5	6	7	8	9	10
41	1.871E-02	1.475E-02	1.072E-02	7.833E-03	5.584E-03	3.740E-03	2.184E-03	8.484E-04	-3.017E-04	-1.283E-03
42	1.645E-02	1.120E-02	8.129E-03	5.837E-03	3.990E-03	2.446E-03	1.134E-03	1.416E-05	-9.333E-04	-1.712E-03
43	1.107E-02	8.153E-03	5.921E-03	4.118E-03	2.615E-03	1.344E-03	2.643E-04	-6.413E-04	-1.378E-03	-1.939E-03
44	7.836E-03	5.806E-03	4.109E-03	2.681E-03	1.470E-03	4.453E-04	-4.086E-04	-1.096E-03	-1.611E-03	-1.940E-03
45	5.495E-03	3.959E-03	2.638E-03	1.509E-03	5.550E-04	-2.360E-04	-8.653E-04	-1.327E-03	-1.606E-03	-1.685E-03
46	3.678E-03	2.490E-03	1.464E-03	5.944E-04	-1.222E-04	-6.840E-04	-1.083E-03	-1.307E-03	-1.336E-03	-1.149E-03
47	2.249E-03	1.340E-03	5.680E-04	-6.319E-05	-5.480E-04	-8.770E-04	-1.036E-03	-1.009E-03	-7.739E-04	-3.094E-04
48	1.149E-03	4.843E-04	-5.194E-05	-4.509E-04	-7.012E-04	-7.888E-04	-6.970E-04	-4.075E-04	9.825E-05	8.355E-04
49	3.555E-04	-7.796E-05	-3.829E-04	-5.468E-04	-5.552E-04	-3.929E-04	-4.408E-05	5.047E-04	1.261E-03	2.219E-03
50	-1.272E-04	-3.313E-04	-4.019E-04	-3.252E-04	-8.788E-05	3.219E-04	9.112E-04	1.678E-03	2.608E-03	3.654E-03
51	-2.799E-04	-2.522E-04	-8.676E-05	2.265E-04	6.940E-04	1.315E-03	2.080E-03	2.965E-03	3.930E-03	4.928E-03
52	-8.184E-05	1.716E-04	5.559E-04	1.071E-03	1.710E-03	2.454E-03	3.276E-03	4.140E-03	5.003E-03	5.828E-03
53	4.569E-04	8.996E-04	1.445E-03	2.079E-03	2.781E-03	3.523E-03	4.274E-03	5.003E-03	5.680E-03	6.284E-03
54	1.245E-03	1.795E-03	2.399E-03	3.036E-03	3.682E-03	4.312E-03	4.905E-03	5.440E-03	5.904E-03	6.285E-03
55	2.088E-03	2.634E-03	3.185E-03	3.721E-03	4.226E-03	4.685E-03	5.086E-03	5.420E-03	5.682E-03	5.869E-03
56	2.737E-03	3.185E-03	3.604E-03	3.985E-03	4.319E-03	4.601E-03	4.825E-03	4.992E-03	5.102E-03	5.158E-03
57	2.996E-03	3.303E-03	3.574E-03	3.804E-03	3.992E-03	4.139E-03	4.246E-03	4.317E-03	4.357E-03	4.372E-03
58	5.602E-03	5.888E-03	6.181E-03	6.476E-03	6.766E-03	7.046E-03	7.310E-03	7.555E-03	7.776E-03	7.973E-03
59	5.841E-03	6.088E-03	6.329E-03	6.559E-03	6.778E-03	6.981E-03	7.169E-03	7.340E-03	7.493E-03	7.631E-03
60	5.996E-03	6.218E-03	6.424E-03	6.613E-03	6.785E-03	6.940E-03	7.077E-03	7.200E-03	7.310E-03	7.410E-03

FIGURE 4a (Cont.)

VALUES OF US/WO AFTER ITR = 1, ITK = 360 AT RHO(2) = 1.0000 US/WO CRIT = 9.774E-02

K	L= 1	2	3	4	5	6	7	8	9	10
2	-7.735E-03	-7.722E-03	-7.650E-03	-7.520E-03	-7.340E-03	-7.118E-03	-6.863E-03	-6.588E-03	-6.306E-03	-6.030E-03
3	-8.356E-03	-7.724E-03	-7.024E-03	-6.291E-03	-5.557E-03	-4.858E-03	-4.226E-03	-3.693E-03	-3.282E-03	-3.015E-03
4	-5.503E-03	-4.210E-03	-3.014E-03	-1.972E-03	-1.136E-03	-5.465E-04	-2.314E-04	-2.060E-04	-4.709E-04	-1.013E-03
5	1.721E-04	1.449E-03	2.353E-03	2.838E-03	2.879E-03	2.478E-03	1.657E-03	4.585E-04	-1.062E-03	-2.835E-03
6	5.982E-03	6.320E-03	6.036E-03	5.153E-03	3.723E-03	1.826E-03	-4.480E-04	-2.995E-03	-5.715E-03	-8.497E-03
7	9.435E-03	8.108E-03	6.098E-03	3.525E-03	5.224E-04	-2.778E-03	-6.251E-03	-9.773E-03	-1.322E-02	-1.646E-02
8	9.098E-03	5.910E-03	2.224E-03	-1.792E-03	-5.990E-03	-1.025E-02	-1.444E-02	-1.845E-02	-2.210E-02	-2.519E-02
9	4.914E-03	2.265E-04	-4.663E-03	-9.626E-03	-1.456E-02	-1.937E-02	-2.391E-02	-2.797E-02	-3.127E-02	-3.345E-02
10	-2.104E-03	-7.732E-03	-1.336E-02	-1.892E-02	-2.435E-02	-2.950E-02	-3.410E-02	-3.766E-02	-3.972E-02	-3.976E-02
11	-1.074E-02	-1.696E-02	-2.315E-02	-2.928E-02	-3.519E-02	-4.051E-02	-4.459E-02	-4.654E-02	-4.570E-02	-4.211E-02
12	-2.028E-02	-2.711E-02	-3.402E-02	-4.089E-02	-4.728E-02	-5.218E-02	-5.419E-02	-5.224E-02	-4.670E-02	-3.872E-02
13	-3.063E-02	-3.840E-02	-4.647E-02	-5.437E-02	-6.070E-02	-6.309E-02	-5.970E-02	-5.163E-02	-4.093E-02	-2.927E-02
14	-4.221E-02	-5.165E-02	-6.162E-02	-7.036E-02	-7.387E-02	-6.853E-02	-5.702E-02	-4.302E-02	-2.871E-02	-1.511E-02
15	-5.596E-02	-6.849E-02	-8.123E-02	-8.759E-02	-7.934E-02	-6.301E-02	-4.492E-02	-2.769E-02	-1.210E-02	1.695E-03
16	-7.387E-02	-9.261E-02	-1.062E-01	-9.315E-02	-6.961E-02	-4.647E-02	-2.609E-02	-8.549E-03	6.479E-03	1.930E-02
17	-1.009E-01	-1.337E-01	-1.116E-01	-7.661E-02	-4.732E-02	-2.370E-02	-4.348E-03	1.174E-02	2.518E-02	3.648E-02
18	-1.803E-01	-1.377E-01	-8.310E-02	-4.685E-02	-2.023E-02	6.338E-04	1.755E-02	3.143E-02	4.302E-02	5.244E-02
19	-1.772E-01	-8.626E-02	-4.399E-02	-1.529E-02	6.555E-03	2.398E-02	3.806E-02	4.983E-02	5.931E-02	6.656E-02
20	-7.742E-02	-3.726E-02	-8.450E-03	1.358E-02	3.111E-02	4.510E-02	5.694E-02	6.640E-02	7.350E-02	7.895E-02
21	-2.561E-02	6.198E-04	2.185E-02	3.901E-02	5.259E-02	6.438E-02	7.372E-02	8.057E-02	8.597E-02	9.016E-02
22	1.194E-02	3.138E-02	4.768E-02	6.048E-02	7.210E-02	8.123E-02	8.776E-02	9.337E-02	9.784E-02	1.006E-01
23	4.218E-02	5.716E-02	6.879E-02	8.011E-02	8.898E-02	9.540E-02	1.011E-01	1.056E-01	1.090E-01	1.078E-01
24	6.747E-02	7.758E-02	8.850E-02	9.741E-02	1.035E-01	1.093E-01	1.142E-01	1.174E-01	1.174E-01	1.014E-01
25	8.743E-02	9.763E-02	1.065E-01	1.121E-01	1.178E-01	1.225E-01	1.258E-01	1.269E-01	1.133E-01	7.518E-02
26	1.074E-01	1.161E-01	1.211E-01	1.265E-01	1.311E-01	1.344E-01	1.363E-01	1.260E-01	8.400E-02	4.795E-02
27	1.259E-01	1.304E-01	1.354E-01	1.398E-01	1.431E-01	1.457E-01	1.383E-01	9.453E-02	5.146E-02	3.103E-02
28	1.400E-01	1.446E-01	1.488E-01	1.520E-01	1.549E-01	1.513E-01	1.071E-01	5.563E-02	3.355E-02	1.724E-02
29	1.541E-01	1.580E-01	1.611E-01	1.641E-01	1.632E-01	1.214E-01	6.106E-02	3.597E-02	1.943E-02	4.501E-03
30	1.675E-01	1.704E-01	1.734E-01	1.745E-01	1.370E-01	6.800E-02	3.795E-02	2.185E-02	6.102E-03	-7.768E-03

FIGURE 4b

VALUES OF US/WO AFTER ITR = 1, ITK = 360 AT RHO(2) = 1.0000 US/WO CRIT = 9.774E-02 (Cont.)

K	L= 1	2	3	4	5	6	7	8	9	10
31	1.799E-01	1.828E-01	1.852E-01	1.530E-01	7.626E-02	3.951E-02	2.459E-02	7.991E-03	-7.153E-03	-1.929E-02
32	1.925E-01	1.953E-01	1.683E-01	8.543E-02	4.101E-02	2.776E-02	1.027E-02	-6.394E-03	-1.992E-02	-2.947E-02
33	2.049E-01	1.815E-01	9.502E-02	4.294E-02	3.154E-02	1.307E-02	-5.398E-03	-2.068E-02	-3.141E-02	-3.764E-02
34	1.912E-01	1.046E-01	4.593E-02	3.613E-02	1.655E-02	-4.038E-03	-2.153E-02	-3.385E-02	-4.071E-02	-4.324E-02
35	1.132E-01	5.081E-02	4.171E-02	2.086E-02	-2.150E-03	-2.239E-02	-3.685E-02	-4.461E-02	-4.695E-02	-4.602E-02
36	5.855E-02	4.827E-02	2.615E-02	4.709E-04	-2.313E-02	-4.048E-02	-4.958E-02	-5.166E-02	-4.978E-02	-4.613E-02
37	5.635E-02	3.257E-02	4.069E-03	-2.352E-02	-4.482E-02	-5.591E-02	-5.763E-02	-5.443E-02	-4.946E-02	-4.398E-02
38	4.019E-02	8.911E-03	-2.320E-02	-4.989E-02	-6.414E-02	-6.523E-02	-6.011E-02	-5.340E-02	-4.561E-02	-4.001E-02
39	1.524E-02	-2.155E-02	-5.554E-02	-7.509E-02	-7.506E-02	-6.697E-02	-5.794E-02	-4.956E-02	-4.190E-02	-3.466E-02
40	-1.763E-02	-6.103E-02	-9.031E-02	-8.801E-02	-7.515E-02	-6.298E-02	-5.270E-02	-4.385E-02	-3.587E-02	-2.822E-02
41	-6.367E-02	-1.130E-01	-1.055E-01	-8.457E-02	-6.828E-02	-5.581E-02	-4.571E-02	-3.698E-02	-2.890E-02	-2.093E-02
42	-1.506E-01	-1.300E-01	-9.463E-02	-7.326E-02	-5.853E-02	-4.726E-02	-3.784E-02	-2.937E-02	-2.122E-02	-1.294E-02
43	-1.647E-01	-1.031E-01	-7.687E-02	-6.037E-02	-4.819E-02	-3.827E-02	-2.951E-02	-2.123E-02	-1.296E-02	-4.360E-03
44	-1.031E-01	-7.752E-02	-6.068E-02	-4.821E-02	-3.808E-02	-2.919E-02	-2.088E-02	-1.267E-02	-4.220E-03	4.697E-03
45	-7.398E-02	-5.899E-02	-4.706E-02	-3.711E-02	-2.832E-02	-2.009E-02	-1.200E-02	-3.749E-03	4.890E-03	1.406E-02
46	-5.533E-02	-4.464E-02	-3.528E-02	-2.681E-02	-1.880E-02	-1.093E-02	-2.917E-03	5.416E-03	1.419E-02	2.346E-02
47	-4.112E-02	-3.261E-02	-2.465E-02	-1.699E-02	-9.413E-03	-1.703E-03	6.281E-03	1.462E-02	2.332E-02	3.227E-02
48	-2.920E-02	-2.186E-02	-1.466E-02	-7.456E-03	-1.112E-04	7.464E-03	1.530E-02	2.334E-02	3.137E-02	3.899E-02
49	-1.854E-02	-1.185E-02	-5.084E-03	1.825E-03	8.910E-03	1.614E-02	2.336E-02	3.031E-02	3.646E-02	4.109E-02
50	-8.634E-03	-2.367E-03	4.025E-03	1.051E-02	1.698E-02	2.322E-02	2.887E-02	3.343E-02	3.628E-02	3.681E-02
51	5.715E-04	6.337E-03	1.207E-02	1.760E-02	2.265E-02	2.687E-02	2.980E-02	3.101E-02	3.019E-02	2.727E-02
52	8.519E-03	1.332E-02	1.771E-02	2.142E-02	2.414E-02	2.553E-02	2.537E-02	2.356E-02	2.023E-02	1.569E-02
53	1.390E-02	1.701E-02	1.931E-02	2.059E-02	2.066E-02	1.947E-02	1.706E-02	1.362E-02	9.432E-03	4.785E-03
54	1.524E-02	1.620E-02	1.625E-02	1.533E-02	1.349E-02	1.085E-02	7.577E-03	3.900E-03	2.060E-05	-3.881E-03

FIGURE 4b (Cont.)

59	1.150E-03	9.608E-04	7.069E-04	4.006E-04	5.646E-05	-3.089E-04	-6.732E-04	-1.034E-03	-1.360E-03	-1.642E-03

VALUES OF UN/WO AFTER ITR = 1, ITK = 360 AT RHO(2) = 1.0000

K	L= 1	2	3	4	5	6	7	8	9	10
2	7.503E-03	7.820E-03	8.224E-03	8.545E-03	8.767E-03	8.880E-03	8.879E-03	8.764E-03	8.538E-03	8.213E-03
3	9.831E-03	9.775E-03	9.765E-03	9.594E-03	9.268E-03	8.800E-03	8.212E-03	7.532E-03	6.791E-03	6.027E-03
4	1.091E-02	1.018E-02	9.511E-03	8.689E-03	7.751E-03	6.742E-03	5.715E-03	4.718E-03	3.801E-03	3.004E-03
5	1.004E-02	8.616E-03	7.340E-03	6.019E-03	4.723E-03	3.516E-03	2.455E-03	1.584E-03	9.364E-04	5.331E-04
6	7.408E-03	5.504E-03	3.924E-03	2.488E-03	1.261E-03	2.894E-04	-3.994E-04	-7.893E-04	-8.720E-04	-6.413E-04
7	3.653E-03	1.693E-03	2.448E-04	-8.366E-04	-1.678E-03	-2.122E-03	-2.216E-03	-1.958E-03	-1.339E-03	-3.481E-04
8	-3.205E-04	-1.911E-03	-2.883E-03	-3.458E-03	-3.646E-03	-3.449E-03	-2.859E-03	-1.855E-03	-4.055E-04	1.507E-03
9	-3.755E-03	-4.746E-03	-5.109E-03	-5.061E-03	-4.600E-03	-3.704E-03	-2.322E-03	-3.925E-04	2.125E-03	5.184E-03
10	-6.328E-03	-6.696E-03	-6.449E-03	-5.768E-03	-4.603E-03	-2.862E-03	-4.325E-04	2.770E-03	6.678E-03	1.095E-02
11	-8.078E-03	-7.880E-03	-7.035E-03	-5.658E-03	-3.601E-03	-6.637E-04	3.322E-03	8.277E-03	1.364E-02	1.847E-02
12	-9.189E-03	-8.465E-03	-6.962E-03	-4.669E-03	-1.256E-03	3.614E-03	9.901E-03	1.669E-02	2.238E-02	2.636E-02
13	-9.864E-03	-8.584E-03	-6.198E-03	-2.423E-03	3.393E-03	1.142E-02	2.020E-02	2.688E-02	3.097E-02	3.285E-02
14	-1.030E-02	-8.298E-03	-4.429E-03	2.244E-03	1.257E-02	2.437E-02	3.216E-02	3.614E-02	3.739E-02	3.681E-02
15	-1.073E-02	-7.529E-03	-5.044E-04	1.271E-02	2.962E-02	3.852E-02	4.198E-02	4.228E-02	4.070E-02	3.798E-02
16	-1.150E-02	-5.799E-03	1.012E-02	3.693E-02	4.643E-02	4.865E-02	4.754E-02	4.474E-02	4.100E-02	3.672E-02
17	-1.365E-02	-5.561E-04	4.886E-02	5.659E-02	5.628E-02	5.319E-02	4.890E-02	4.404E-02	3.888E-02	3.361E-02
18	4.374E-03	7.314E-02	6.980E-02	6.492E-02	5.918E-02	5.316E-02	4.707E-02	4.100E-02	3.507E-02	2.907E-02
19	1.059E-01	8.605E-02	7.443E-02	6.542E-02	5.746E-02	5.007E-02	4.307E-02	3.651E-02	2.994E-02	2.358E-02
20	1.046E-01	8.432E-02	7.172E-02	6.170E-02	5.299E-02	4.505E-02	3.791E-02	3.078E-02	2.404E-02	1.758E-02
21	9.417E-02	7.781E-02	6.579E-02	5.577E-02	4.692E-02	3.925E-02	3.157E-02	2.449E-02	1.777E-02	1.105E-02
22	8.358E-02	6.966E-02	5.842E-02	4.870E-02	4.058E-02	3.235E-02	2.497E-02	1.801E-02	1.078E-02	3.388E-03
23	7.330E-02	6.088E-02	5.036E-02	4.187E-02	3.312E-02	2.547E-02	1.791E-02	1.026E-02	2.804E-03	-5.337E-03
24	6.314E-02	5.183E-02	4.302E-02	3.379E-02	2.553E-02	1.748E-02	9.828E-03	2.257E-03	-5.793E-03	-1.660E-02
25	5.322E-02	4.342E-02	3.395E-02	2.522E-02	1.710E-02	9.338E-03	1.634E-03	-6.327E-03	-1.682E-02	-3.034E-02
26	4.317E-02	3.373E-02	2.489E-02	1.660E-02	8.710E-03	9.094E-04	-6.982E-03	-1.699E-02	-3.228E-02	-3.547E-02
27	3.362E-02	2.438E-02	1.594E-02	7.927E-03	5.418E-05	-7.792E-03	-1.723E-02	-3.318E-02	-3.835E-02	-3.861E-02
28	2.389E-02	1.510E-02	6.966E-03	-9.597E-04	-8.780E-03	-1.767E-02	-3.307E-02	-4.171E-02	-4.116E-02	-4.113E-02
29	1.428E-02	5.805E-03	-2.158E-03	-9.967E-03	-1.839E-02	-3.258E-02	-4.531E-02	-4.370E-02	-4.375E-02	-4.315E-02
30	4.694E-03	-3.562E-03	-1.137E-02	-1.944E-02	-3.218E-02	-4.843E-02	-4.612E-02	-4.641E-02	-4.601E-02	-4.434E-02

FIGURE 4c

59	1.150E-03	9.608E-04	7.069E-04	4.006E-04	5.646E-05	-3.089E-04	-6.782E-04	-1.034E-03	-1.360E-03	-1.642E-03

VALUES OF UN/WO AFTER ITR = 1, ITK = 360 AT RHO(2) = 1.0000

K	L= 1	2	3	4	5	6	7	8	9	10
31	-4.886E-03	-1.299E-02	-2.083E-02	-3.205E-02	-5.056E-02	-4.833E-02	-4.908E-02	-4.903E-02	-4.740E-02	-4.444E-0
32	-1.451E-02	-2.256E-02	-3.229E-02	-5.134E-02	-5.019E-02	-5.171E-02	-5.222E-02	-5.076E-02	-4.753E-02	-4.333E-0
33	-2.420E-02	-3.293E-02	-5.036E-02	-5.171E-02	-5.427E-02	-5.554E-02	-5.443E-02	-5.102E-02	-4.617E-02	-4.108E-0
34	-3.354E-02	-4.720E-02	-5.302E-02	-5.672E-02	-5.899E-02	-5.846E-02	-5.499E-02	-4.945E-02	-4.333E-02	-3.797E-0
35	-4.269E-02	-5.373E-02	-5.904E-02	-6.255E-02	-6.285E-02	-5.953E-02	-5.327E-02	-4.591E-02	-3.933E-02	-3.448E-0
36	-5.366E-02	-6.122E-02	-6.617E-02	-6.763E-02	-6.475E-02	-5.782E-02	-4.891E-02	-4.078E-02	-3.488E-02	-3.108E-0
37	-6.315E-02	-6.985E-02	-7.278E-02	-7.080E-02	-6.336E-02	-5.251E-02	-4.226E-02	-3.513E-02	-3.064E-02	-2.807E-0
38	-7.333E-02	-7.825E-02	-7.779E-02	-7.028E-02	-5.698E-02	-4.368E-02	-3.507E-02	-2.992E-02	-2.701E-02	-2.557E-0
39	-8.365E-02	-8.576E-02	-7.919E-02	-6.292E-02	-4.476E-02	-3.445E-02	-2.875E-02	-2.565E-02	-2.410E-02	-2.354E-0
40	-9.404E-02	-9.088E-02	-7.176E-02	-4.483E-02	-3.283E-02	-2.693E-02	-2.387E-02	-2.238E-02	-2.182E-02	-2.185E-0
41	-1.041E-01	-8.750E-02	-4.199E-02	-2.951E-02	-2.417E-02	-2.158E-02	-2.036E-02	-1.994E-02	-2.000E-02	-2.033E-0
42	-1.111E-01	-3.030E-02	-2.350E-02	-2.026E-02	-1.869E-02	-1.802E-02	-1.788E-02	-1.807E-02	-1.843E-02	-1.880E-0
43	-1.105E-02	-1.456E-02	-1.517E-02	-1.523E-02	-1.536E-02	-1.565E-02	-1.605E-02	-1.650E-02	-1.689E-02	-1.710E-0
44	-5.488E-03	-9.445E-03	-1.141E-02	-1.250E-02	-1.329E-02	-1.397E-02	-1.454E-02	-1.498E-02	-1.519E-02	-1.507E-0
45	-4.781E-03	-7.588E-03	-9.589E-03	-1.090E-02	-1.186E-02	-1.258E-02	-1.308E-02	-1.329E-02	-1.315E-02	-1.255E-0
46	-4.882E-03	-6.829E-03	-8.569E-03	-9.732E-03	-1.064E-02	-1.119E-02	-1.141E-02	-1.124E-02	-1.059E-02	-9.372E-0
47	-5.058E-03	-6.405E-03	-7.796E-03	-8.749E-03	-9.328E-03	-9.533E-03	-9.320E-03	-8.619E-03	-7.339E-03	-5.379E-0
48	-5.095E-03	-5.955E-03	-6.943E-03	-7.514E-03	-7.676E-03	-7.336E-03	-6.614E-03	-5.249E-03	-3.203E-03	-3.593E-0
49	-4.844E-03	-5.236E-03	-5.750E-03	-5.832E-03	-5.454E-03	-4.562E-03	-3.083E-03	-9.283E-04	1.999E-03	5.776E-0
50	-4.148E-03	-4.031E-03	-3.988E-03	-3.475E-03	-2.441E-03	-8.241E-04	1.448E-03	4.431E-03	8.122E-03	1.237E-0
51	-2.826E-03	-2.118E-03	-1.430E-03	-2.274E-04	1.540E-03	3.910E-03	6.879E-03	1.036E-02	1.411E-02	1.779E-0
52	-6.715E-04	7.158E-04	2.102E-03	4.003E-03	6.408E-03	9.250E-03	1.237E-02	1.552E-02	1.841E-02	2.080E-0
53	2.430E-03	4.549E-03	6.519E-03	8.849E-03	1.142E-02	1.405E-02	1.653E-02	1.866E-02	2.029E-02	2.140E-0
54	6.505E-03	8.984E-03	1.109E-02	1.324E-02	1.529E-02	1.707E-02	1.848E-02	1.945E-02	1.998E-02	2.008E-0
55	1.063E-02	1.294E-02	1.455E-02	1.596E-02	1.706E-02	1.781E-02	1.818E-02	1.820E-02	1.786E-02	1.720E-0
56	1.358E-02	1.517E-02	1.592E-02	1.639E-02	1.655E-02	1.642E-02	1.600E-02	1.532E-02	1.440E-02	1.328E-0

FIGURE 4c (Cont.)

NOTE: THE FOLLOWING VALUES OF V RADIAL ARE BASED ON THE 2D STRIP-THEORY APPROXIMATION

VALUES OF V RADIAL/U INFINITYAFTER ITR = 1, ITK = 360 AT RHO(2) = 1.0000

K	L= 1	2	3	4	5	6	7	8	9	10
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
22	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
24	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
26	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
27	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
28	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
29	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

FIGURE 4d

NOTE: THE FOLLOWING VALUES OF V RADIAL ARE BASED ON THE 2D STRIP-THEORY APPROXIMATION (Cont.)

VALUES OF V RADIAL/U INFINITYAFTER ITR = 1, ITK = 360 AT RHO(2) = 1.0000

K	L= 1	2	3	4	5	6	7	8	9	10
31	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
32	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
33	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
34	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
35	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
36	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
37	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
38	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
39	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
41	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
42	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
43	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
44	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
45	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
46	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
47	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
48	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
49	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
51	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
52	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
53	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
54	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
55	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

FIGURE 4d (cont.)

```
TRAILING-EDGE VELOCITIES ARE US(1,TE) = -1.638E-01  US(LMX,TE) = -1.695E-01  DELTA US =  5.776E-03  DPHI( 2) = -5.134E-03

AFTER ITR =  1 MAXIMUM RESIDUALS ARE
```

N	RESIDUAL	K	L	AVG RES	AVG PHI	AVG SUM
2	2.281E-04	37	29	1.672E-05	6.098E-03	3.193E-03

PERFORMANCE DATA

N	RHO(N)	DPHI(N)	W/U INF	U/U INF	ARCTAN(UN/WO) (DEGREES)	P02/P01
1	0.9474	-5.134E-03	0.0086	0.0056	0.10	0.996
2	1.0000	-5.134E-03	0.0082	0.0056	0.07	0.996
3	1.0526	-5.134E-03	0.0078	0.0056	0.05	0.996

FIGURE 5

APPENDIX A

GUIDE TO PREPARING THE INPUT

Cards 1 and 2: FORMAT 18A4: These cards are for run identification; any alphameric characters may be used. Neither card can be omitted.

Card 3: FORMAT 5E15:

Columns 1-15: B, the number of blades. For a two-dimensional cascade solution, use B=10 (and see comments below regarding IDM=2).

Columns 16-30: h, the hub-to-tip radius ratio. For a two-dimensional cascade solution, use h = 0.9.

Columns 31-35: Solidity, $C_a / L_T = C_a / \frac{2\pi r_T}{B}$

Columns 46-60: Axial Mach number at the inlet, U_∞ / a_∞

Columns 61-75: Tangential Mach number at the tip, $\omega r_T / a_\infty$. The present version of the program was developed for inlet relative Mach numbers $\sqrt{U_\infty^2 + (\omega r_T)^2} / a_\infty$ less than one. The program cannot handle values greater than one, because the boundary conditions imposed at the upstream and downstream edges of the grid are not radiation conditions.

Card 4: FORMAT 5E15

Columns 1-15: γ, the specific-heat ratio.

Columns 16-30: FXB: The upstream edge of the grid will be located at FXB times the axial projection of the chord C_a. Values of FXB = -2 or -3 have been used. FXB must be negative.

Columns 31-45: FXI: The downstream edge of the grid will be located at FXI • C_a downstream of the trailing edge. FXI = -FXB will place the greatest concentration of grid points near the mid-chord of the blades.

Columns 46-60: RXE is the relaxation factor used at subsonic points. Values up to 1.2 (and "tapered", i.e., ITPR=1 - see below) have been used for the off-design problem (IBC=1), but very little is known about the limitations on this parameter. For the design problem (IBC=2), it may be necessary to use values as low as 0.1, with ITPR=0, early in the

iterations, increasing to 0.8 (and ITPR=0) at the later stages. Again, very little is known about these limitations.

Columns 61-75: RXH is the relaxation factor used at supersonic points. RXH=0.9 has been used successfully. The ITPR parameter does not affect RXH - a constant value is used, at all supersonic points in the grid.

Card 5: FORMAT 5E15

Columns 1-15: OBV, the relaxation factor used in updating the circulation. Values of 0.05 and 0.1 have been used, but the limits on this parameter and their coupling to the other relaxation factors are unknown.

Card 6: FORMAT 16I5

Columns 1-5/6-10/11-15: Grid size parameters KMX/LMX/NMX. These are limited to 60/30/10, by a COMMON statement in the version listed below. The number of calculations done in a given z-sweep is proportional to KMX, and the number of z-sweeps required for convergence is also proportional to KMX (since it takes KMX iterations before information at the downstream edge of the grid affects the solution at the upstream edge). Thus, the total time required for a given calculation is likely to vary as the square of KMX, and values as low as possible should be used.

If shock capturing is important, LMX should then be chosen so as to make $\rho\Delta\zeta/\Delta z$ approximately equal to $(1+\rho^2)/\rho$ at a radius in the center of the region where shock definition is desired. For the particular stretching of the z-coordinates used here, and for the special case where FXI = -FXB, the value of $\rho\Delta\zeta/\Delta z$ at midchord is

$$\left.\frac{\rho\Delta\zeta}{\Delta z}\right)_{x=c_a/2} = \frac{\rho}{\rho_T}\cdot\frac{L_T}{c_a}\cdot\frac{1}{2(1+2\,FXI)}\cdot\frac{(KMX+1)\ln KMX}{LMX-1} \tag{A-1}$$

This equation can be solved for LMX, once the other parameters are chosen.

The quantity NMX must be three or greater; for a two-dimensional cascade solution, set it equal to 3.

Column 20: IBC=1 for the off-design case, IBC=2 for the design case.

Column 25: IDM=3 for a three-dimensional solution. For a two-dimensional cascade solution, set IDM=2 and NMX=3. For this case, the hub/tip ratio h and the number of blades B are meaningless, and can be read in as any arbitrary numbers; the values h = 0.9 and B = 10 are recommended. The desired cascade parameters are actually the inlet relative Mach number M_0 and the stagger angle. These determine ρ, which is the ratio of the pitchwise to axial velocity components upstream of the cascade:

$$\rho = \omega r / u_\infty$$

The hub and tip radii are then determined by the fact that, for NMX=3, the value of ρ lies half-way between ρ_H and ρ_T :

$$\rho = \frac{\rho_H + \rho_T}{2} = \frac{h+1}{2}\,\rho_T\ ;$$

Thus,

$$\rho_T = \frac{2}{1+h}\,\rho\ , \quad \rho_H = \frac{2h}{1+h}\,\rho \tag{A-2}$$

The desired value of the pitchwise component of the inlet Mach number at station ρ is

$$M_\theta = \rho M_x = \frac{\rho}{\rho_T} M_{\theta,tip}$$

$$= \frac{\rho}{\rho_T}\cdot\rho_T M_x = \frac{\rho_T}{\sqrt{1+\rho^2}} M_0 = \frac{2}{1+h}\,\frac{\rho}{\sqrt{1+\rho^2}} M_0 \tag{A-3}$$

For a two-dimensional strip-theory calculation of a three-dimensional case, set IDM=2 and 3 $<$ NMX $\leq$ 10. In the latter case, all radial derivatives are set equal to zero, and each radial station is treated as though it were a two-dimensional section.

Columns 26-30: ITKMX is the maximum number of iterations that will be made in the z-direction, on each radial sweep.

Columns 31-35: JRPT: Output is printed at intervals of JRPT in the z-direction iterations on every radial sweep. It is used only for looking

at intermediate results, and gives a large amount of output. Normally, results are needed only after a given number of radial iterations (see NRPT below). To suppress these intermediate results, set JRPT $>$ ITKMX.

Columns 36-40: ISTART = 0 if you are starting from scratch; ISTART = 1 if you are reading in values of ϕ (on tape) from a previous run.

Columns 41-45: ISAVE=1 will write all values of $\Delta\phi(N)$ and $\phi(L,K,N)$ on tape at the end of the calculation. ISAVE=0 writes no tape.

Columns 46-50: IOP controls what is printed on output. See the comment cards in subroutine OUTPUT for details.

Columns 51-55: ITRMX gives the maximum number of radial iterations. On each radial iteration, ITKMX axial iterations are done, at radial stations N=2, NMX-1.

Columns 56-60: NRPT: Output will occur at intervals of NRPT in the radial iterations.

Columns 61-65: ITPR=1 will cause the relaxation factor used at elliptic points to be "tapered", according to

$$\omega = 1 + (RXE - 1)\ exp\left\{-\left(\frac{Z - \frac{1}{2}\,\frac{\omega c_a}{u_\infty}}{\omega c_a / u_\infty}\right)^2\right\} \qquad \text{(A-4)}$$

ITPR=0 uses ω=RXE at all elliptic points.

Columns 66-70: Residuals are calculated and printed at intervals of IRXP in the radial iterations. If IRXP is preceded by a negative sign, the parameter ISHO is internally set equal to 1, and absolute values of all terms in the potential equation are displayed as well.

Cards 7 and 8: These contain the input data for blade shape and loading - see comments in subroutine BVAL for details. Both cards must be used, with dummy zeros, if necessary.

Card 8: If IOP=4, this card specifies the locations and Mach-number values at which Mach-number contours are calculated.

APPENDIX B

PROGRAM LISTING

```
LEVEL 21.7 ( DEC 72 )                                  OS/360  FORTRAN H

         COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                            SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
                 C
                 C       PROGRAM 3DNSDA - THREE-DIMENSIONAL NONLINEAR SMALL-DISTURBANCE
                 C       (VERSION A) PROGRAM FOR CALCULATING THE FLOW THROUGH AN
                 C       ISOLATED COMPRESSOR BLADE ROW.  DOCUMENTATION IS GIVEN IN:
                 C             W. J. RAE, NONLINEAR SMALL-DISTURBANCE EQUATIONS FOR
                 C                        THREE-DIMENSIONAL TRANSONIC FLOW THROUGH A
                 C                        COMPRESSOR BLADE ROW, CALSPAN CORPORATION
                 C                        REPORT AB-5487-A-1, AFOSR TR-76-1082,
                 C                        AD A031234 (AUG 1976)
                 C             - - -      RELAXATION SOLUTIONS FOR THREE-DIMENSIONAL
                 C                        TRANSONIC FLOW THROUGH A COMPRESSOR BLADE ROW,
                 C                        IN THE NONLINEAR SMALL-DISTURBANCE APPROXIMA-
                 C                        TION, CALSPAN CORPORATION REPORT AB-5487-A-2,
                 C                        AFOSR TR-76-1081, AD A032553 (AUG 1976)
                 C             - - -      CALCULATIONS OF THREE-DIMENSIONAL TRANSONIC
                 C                        COMPRESSOR FLOWFIELDS BY A RELAXATION METHOD,
                 C                        AIAA JOURNAL OF ENERGY, VOL. 1, NO. 5,
                 C                        SEPT - OCT 1977, PP 284 - 296
                 C
                 C         USERS ARE URGED TO COMMUNICATE WITH THE AUTHOR, CONCERNING
                 C         PROBLEMS, RESULTS, AND SUGGESTED CHANGES -
                 C         AERODYNAMIC RESEARCH DEPARTMENT, CALSPAN CORPORATION
                 C         PO BOX 235
                 C         BUFFALO, NY 14221
                 C         TELEPHONE 716/632-7500
                 C
  ISN 0002             COMMON ZT(30),XI(60),RO(10),F(30,60,10),A(10),B(10),D(10),E(10),
                      *   EE(30),FF(30),SV(30),DPHI(10),BV(10),US(30,60),UN(30,60),
                      *   DNDS(30,10,2),FAK(60),FHW(60),UTA(30),UTB(30),UTC(30),BBV(30),
                      *   BSA(30),BSB(30),BSC(30),VSA(30),VSB(30),BAV(30),LOP(30)
                      *   ,SONIC(10)
  ISN 0003             COMMON DAF,DBF,DCF,AKLKE,RX,OX,CONST
  ISN 0004             COMMON H,DRO,DZT,TDZ,TPB,XIC,XIB,XID,TDR,AKLK,DRO2,DZT2
  ISN 0005             COMMON OMCU,RHUB,RTIP,R2,RXE,RXH,UA,UB,TRDZ,ERA,ERB,CKLKE,SB,
                      *   BTR,FXB,FXI,EM2,AMA,AMB,DM,DKA,DKR,DQ,DQS,TZ,TTZ,TZS,TRS,TDQ
  ISN 0006             COMMON K,N,IDM,     KMX,LMX,NMX,KMXM1,KMXM2,LMXM1,LMXM2,NMXM1,
                      *   NMXM2,KTEO,IRX,    ITK,ITR,ITRMX,KLEP,ITPR,KUP,KDN
  ISN 0007             COMMON ID(36)
                 C
                 C      THE FOLLOWING CALSPAN ROUTINE PLACES A ZERO IN THE LOCATIONS
                 C      FROM ZT(1) THROUGH ID(36)
                 C
  ISN 0008             CALL CLEAR(ZT(1),ID(36))
                 C
  ISN 0009             READ(5,102) (ID(I),I=1,36)
  ISN 0010         102 FORMAT(18A4)
  ISN 0011             WRITE(6,200)
  ISN 0012         200 FORMAT(1H1)
  ISN 0013             WRITE(6,201) (ID(I),I=1,36)
  ISN 0014         201 FORMAT(30X,18A4)
  ISN 0015             READ(5,100) BN,H,CALT,EMX,EMTG,GMA,FXB,FXI,RXE,RXH,OBV
  ISN 0016         100 FORMAT(5E15.5)
  ISN 0017             READ(5,101) KMX,LMX,NMX,IBC,IDM,ITKMX,JPRT,ISTART,ISAVE,IOP
                      *   ,ITRMX,NPRT,ITPR,IRXP
  ISN 0018         101 FORMAT(16I5)
  ISN 0019             READ(5,100) (BV(I),I=1,10)
```

```
ISN 0020          IBN = INT(BN+0.01)
ISN 0021          WRITE(6,202) IBN,H,CALT
ISN 0022      202 FORMAT(//5X,'THIS BLADE ROW HAS',I4,' BLADES, WITH A HUB-TO-TIP',
                 *    ' RATIO OF ',F5.3,' AND SOLIDITY CA/LT = ',F5.3)
ISN 0023          RTIP = EMTG/EMX
ISN 0024          RHUB = H*RTIP
ISN 0025          EM2 = EMX*EMX
ISN 0026          REL = SQRT(EM2+EMTG*EMTG)
ISN 0027          WRITE(6,203) EMX,EMTG,REL,GMA
ISN 0028      203 FORMAT( /5X,'AXIAL MACH NO. = ',F5.3,',  TANGENTIAL MACH NO  AT',
                 *    ' THE TIP = ',F5.3,',  TOTAL MACH NO. AT THE TIP = ',F5.3,
                 *    //5X,'SPECIFIC HEAT RATIO =',F6.3)
ISN 0029          IF(REL.GT.1.) WRITE(6,207)
ISN 0031      207 FORMAT(//15X,'WARNING - INLET RELATIVE MACH NUMBER AT THE TIP ',
                 *    'IS SUPERSONIC',
                 *    /15X,'BOUNDARY CONDITIONS UPSTREAM AND DOWNSTREAM DO NOT ',
                 *    'CONTAIN THE REQUIRED RADIATION CONDITIONS',
                 *    /15X,'SOLUTION UPSTREAM MAY BE INVALID',
                 *    /15X,'SOLUTION DOWNSTREAM MAY NOT CONVERGE, ESPECIALLY IF',
                 *    ' THE OUTLET RELATIVE MACH NUMBER IS SUPERSONIC',/)
ISN 0032          IF(ISTART.NE.1) GO TO 98
                C
                C   READ STARTING VALUES FROM TAPE
                C
ISN 0034          KS = KMX
ISN 0035          LS = LMX
ISN 0036          NS = NMX
ISN 0037          READ(1) LMX,KMX,NMX,DPHI
ISN 0038          DO 97 N=1,NMX
ISN 0039          READ(1) ((F(L,K,N),L=1,LMX),K=1,KMX)
ISN 0040       97 CONTINUE
ISN 0041          IF(KS.NE.KMX) GO TO 33
ISN 0043          IF(LS.NE.LMX) GO TO 33
ISN 0045          IF(NS.NE.NMX) GO TO 33
ISN 0047          GO TO 98
ISN 0048       33 WRITE(6,210) KS,LS,NS,KMX,LMX,NMX
ISN 0049      210 FORMAT(//5X,'WARNING - GRID SIZE ON CARD INPUT DIFFERS FROM',
                 *    ' THAT ON TAPE',
                 *    //10X,'CARD KMX = ',I4,' LMX = ',I4,' NMX = ',I4,
                 *    //10X,'TAPE KMX = ',I4,' LMX = ',I4,' NMX = ',I4,)
                C
                C   END OF TAPE INPUT
                C
ISN 0050       98 WRITE(6,204) IDM,KMX,LMX,NMX
ISN 0051      204 FORMAT( /5X,'THIS IS A',I2,'-DIMENSIONAL CALCULATION, WITH GRID',
                 *    ' SIZE KMX/LMX/NMX = ',I3,'/',I3,'/',I3)
ISN 0052          WRITE(6,206)
ISN 0053      206 FORMAT(/5X,'RELAXATION FACTORS FOR ELLIPTIC AND HYPERBOLIC',
                 *    ' POINTS ARE LISTED BELOW AS RXE AND RXH')
ISN 0054          SB = (GMA + 1.)*EM2
ISN 0055          TPI = 2.0*3.14159265
ISN 0056          TPB = TPI/BN
ISN 0057          OMCU = TPB*RTIP*CALT
ISN 0058          LMXM1 = LMX -1
ISN 0059          LMXM2 = LMX - 2
ISN 0060          KMXM1 = KMX - 1
ISN 0061          KMXM2 = KMX -2
ISN 0062          NMXM1 = NMX - 1
ISN 0063          NMXM2 = NMX - 2
ISN 0064          CALL GRID
ISN 0065          DO 99 I = 1,LMX
ISN 0066       99 LOP(I) = I
ISN 0067          IF(IOP.NE.4) GO TO 15
ISN 0069          READ(5,104) AMA,AMB,DM,KUP,KDN
ISN 0070      104 FORMAT(3E15.5,3I5)
```

```
ISN 0071          DO 21 N = 1,NMX
ISN 0072          UA = RO(N)
ISN 0073          UB =(1./(1.+RO(N)*RO(N)))/TDQ
ISN 0074          TRDZ = TDZ*RO(N)
ISN 0075          CALL OUTPUT(IOP,IBC)
ISN 0076       21 CONTINUE
ISN 0077          GO TO 30
            C
ISN 0078       15 CALL BVAL(IBC)
ISN 0079        9 OBM = 1. - OBV
ISN 0080          ITR = 1
ISN 0081          NPT = 0
ISN 0082          IRXPT = 0
ISN 0083          ISHO = 0
ISN 0084          IF(IRXP.GE.0) GO TO 44
ISN 0086          IRXP = -IRXP
ISN 0087          ISHO = 1
ISN 0088       44 BGL = RO(1)*DPHI(1)/2.
ISN 0089          DO 45 N = 2,NMXM1
ISN 0090       45 BGL = BGL + RO(N)*DPHI(N)
ISN 0091          BGL = (BGL + RO(NMX)*DPHI(NMX)/2.)*DRO
ISN 0092          CONST =-2.*BGL/(TPB*(1.-EM2)*(RTIP*RTIP-RHUB*RHUB))
            C
            C    THE FOLLOWING STATEMENTS ARE USED TO SUPPRESS EXTRANEOUS OUTPUT
            C    WHEN THE SOLUTION IS BEING FOUND AT ONLY A SINGLE RADIAL STATION.
            C
ISN 0093          NA = 1
ISN 0094          NB = NMX
ISN 0095          IF(NMX.NE.3) GO TO 304
ISN 0097          NA = 2
ISN 0098          NB = 2
            C
            C   BEGINNING OF RHO - SWEEP
            C
ISN 0099      304 DO 300 N = 2,NMXM1
ISN 0100          R2 = RO(N)*RO(N)
ISN 0101          BTR = 1.0 - EM2*(1.0+R2)
ISN 0102          AKLK = D(N)*TZS
ISN 0103          UA = RO(N)
ISN 0104          UB =(1./(1.+RO(N)*RO(N)))/TDQ
ISN 0105          TRDZ = TDZ*RO(N)
ISN 0106          DKA = -B(N)*TTZ
ISN 0107          ERA = 1. + 0.5*DRO/RO(N)
ISN 0108          ERB = 1. - 0.5*DRO/RO(N)
ISN 0109          ITK = 1
ISN 0110          IPRT = 0
ISN 0111          IF(IDM.NE.2) GO TO 1
ISN 0113          CONST =-DPHI(N)/(TPB*(1.-EM2))
            C
            C   BEGINNING OF XI - SWEEP
            C
ISN 0114        1 DO 2 L = 1,LMX
ISN 0115          UTA(L) = 0.0
ISN 0116          UTB(L) = 0.0
ISN 0117          VSA(L) = 0.
ISN 0118          BAV(L) = BTR
ISN 0119        2 CONTINUE
ISN 0120       38 DO 5 K = 2,KMXM1
ISN 0121          RX = 1. + (RXE-1.)*EXP(-((XI(K)-0.5*XID)/XID)**2)
ISN 0122          IF(ITPR.EQ.0) RX = RXE
ISN 0124          OX = 1. - 1./RX
ISN 0125          RXM = 1.0 - RX
ISN 0126          IRX = 1
```

```
                 C
                 C   BEGINNING OF ZETA - SWEEP
                 C
ISN 0127              4 AKLKE = AKLK/FAK(K)
ISN 0128                DKR = TRS*E(N)/FAK(K)
ISN 0129                CKLKE = AKLKE
ISN 0130             29 DO 3 L = 1,LMX
ISN 0131                UTC(L) = UTB(L)
ISN 0132                UTB(L) = UTA(L)
ISN 0133                UTA(L) = F(L,K,N)
ISN 0134                BBV(L) = BAV(L)
ISN 0135                BAV(L) = BTR - SB*FAK(K)*(F(L,K+1,N)-F(L,K-1,N))/TDQ
ISN 0136                VSB(L) = VSA(L)
ISN 0137                VSA(L) = 0.
ISN 0138                IF(BAV(L).GT.0.) GO TO 8
ISN 0140                VSA(L) = 1.
ISN 0141                IRX = 2
ISN 0142              8 BSA(L) = (1.-VSA(L))*(BAV(L)+A(N))
ISN 0143                BSB(L) = VSB(L)*BBV(L)
ISN 0144                BSC(L) = VSA(L)*A(N)
ISN 0145              3 CONTINUE
                 C
                 C   UTA,UTB, AND UTC NOW CONTAIN DATA, FROM THE PREVIOUS ITERATION,
                 C   FOR F(L,K,N),F(L,K-1,N), AND F(L,K-2,N), RESPECTIVELY
                 C
ISN 0146             14 CALL BVAL0(IBC)
ISN 0147                DO 7 L = 2,LMXM1
ISN 0148                BKL = -2.*AKLKE - 2.*FAK(K)*BSA(L)/RX + 2.*FHW(K-1)*BSB(L)
                      *    - BSC(L)*FHW(K-1)
ISN 0149                DKL = -BSA(L)*(FHW(K)*F(L,K+1,N)-2.*FAK(K)*OX*UTA(L)
                      *    + FHW(K-1)*F(L,K-1,N)) + BSB(L)*(FHW(K-1)*(UTA(L)+F(L,K-1,N))
                      *    + FHW(K-2)*(F(L,K-1,N)-UTC(L))) - BSC(L)*(FHW(K)*(F(L,K+1,N)
                      *    -UTA(L)) + FHW(K-1)*F(L,K-1,N))
                      *    + 2.*DKA*(-F(L+1,K-1,N)+F(L,K-1,N)+F(L+1,K,N)-2.*F(L,K,N)
                      *    + F(L-1,K,N)+F(L,K+1,N)-F(L-1,K+1,N))
                      *    -DKR*(ERA*F(L,K,N+1)+ERB*F(L,K,N-1)-2.*UTA(L))
ISN 0150                DNM = BKL + CKLKE*EE(L-1)
ISN 0151                EE(L) = -AKLKE/DNM
ISN 0152             16 FF(L) = (DKL - CKLKE*FF(L-1))/DNM
ISN 0153              7 CONTINUE
ISN 0154                CALL BVAL1(IBC)
                 C
ISN 0155            406 DO 19 J = 1,LMXM1
ISN 0156                L = LMX-J
ISN 0157                SV(L) = EE(L)*SV(L+1) + FF(L)
ISN 0158             19 CONTINUE
                 C
ISN 0159             70 IF(IRX.EQ.1) GO TO 6
ISN 0161             11 IF(IRX.EQ.2) RX = RXH
ISN 0163                RXM = 1.0 - RX
                 C
ISN 0164              6 DO 12 L = 1,LMX
ISN 0165             12 F(L,K,N) = RX*SV(L) + RXM*F(L,K,N)
                 C
ISN 0166              5 CONTINUE
                 C
ISN 0167                DO 41 L = 1,LMX
ISN 0168             41 F(L,KMX,N) = (CONST-DAF*F(L,KMXM2,N)-DBF*F(L,KMXM1,N))/DCF
ISN 0169            307 IF(IBC.EQ.2) GO TO 43
                 C
                 C   RESET DPHI
                 C
ISN 0171             47 X1 = XI(KTE0-2)
ISN 0172                X2 = XI(KTE0-1)
ISN 0173                X3 = XID
ISN 0174                CD = (X2-X1)*(2.*X3-X1-X2)
```

```
ISN 0175          EBD = -(X3-X2)*(X3-X2)/CD
ISN 0176          FBD = (X3-X1)*(X3-X1)/CD
ISN 0177          DFY = EBD*(F(1,KTEO-2,N) - F(LMX,KTEO-2,N))
                 *   + FBD*(F(1,KTEO-1,N) - F(LMX,KTEO-1,N))
ISN 0178          DPHI(N) = OBV*DFY + OBM*DPHI(N)
            C
ISN 0179       43 IPRT = IPRT + 1
ISN 0180          IF(IPRT.LT.JPRT) GO TO 42
ISN 0182          IPRT = 0
ISN 0183          CALL OUTPUT(IOP,IBC)
ISN 0184       42 ITK = ITK + 1
ISN 0185          IF(ITK.GT.ITKMX) GO TO 300
ISN 0187          GO TO 1
ISN 0188      300 CONTINUE
            C
ISN 0189      306 IF(NMX.EQ.3) GO TO 310
ISN 0191          DO 301 K = 1,KMX
ISN 0192          DO 302 L = 1,LMX
ISN 0193          F(L,K,1) = (4.*F(L,K,2)-F(L,K,3))/3.
ISN 0194          F(L,K,NMX) = (4.*F(L,K,NMXM1)-F(L,K,NMXM2))/3.
ISN 0195      302 CONTINUE
ISN 0196      301 CONTINUE
ISN 0197          DPHI(1) = 2.*DPHI(2) - DPHI(3)
ISN 0198          DPHI(NMX) = 2.*DPHI(NMXM1) - DPHI(NMXM2)
ISN 0199          GO TO 305
ISN 0200      310 DO 311 K = 1,KMX
ISN 0201          DO 312 L = 1,LMX
ISN 0202          F(L,K,1) = F(L,K,2)
ISN 0203          F(L,K,3) = F(L,K,2)
ISN 0204      312 CONTINUE
ISN 0205      311 CONTINUE
ISN 0206          DPHI(1) = DPHI(2)
ISN 0207          DPHI(3) = DPHI(2)
ISN 0208      305 NPT = NPT + 1
ISN 0209          IF(NPT.LT.NPRT) GO TO 62
ISN 0211          NPT = 0
ISN 0212          ITK = ITK - 1
ISN 0213          DO 63 N = NA,NB
ISN 0214          UA = RO(N)
ISN 0215          UB =(1./(1.+RO(N)*RO(N)))/TDQ
ISN 0216          TRDZ = TDZ*RO(N)
ISN 0217          CALL OUTPUT(IOP,IBC)
ISN 0218       63 CONTINUE
ISN 0219       62 CONTINUE
ISN 0220          IRXPT = IRXPT + 1
ISN 0221          IF(IRXPT.NE.IRXP) GO TO 32
ISN 0223          IRXPT = 0
ISN 0224          CALL RESID(ISHO)
ISN 0225       32 CONTINUE
ISN 0226          IF(IBC.EQ.2) GO TO 34
ISN 0228          BGL = RO(1)*DPHI(1)/2.
ISN 0229          DO 46 N = 2,NMXM1
ISN 0230       46 BGL = BGL + RO(N)*DPHI(N)
ISN 0231          BGL = (BGL + RO(NMX)*DPHI(NMX)/2.)*DRO
ISN 0232          CONST =-2.*BGL/(TPB*(1.-EM2)*(RTIP*RTIP-RHUB*RHUB))
ISN 0233       34 CONTINUE
ISN 0234          ITR = ITR + 1
ISN 0235          IF(ITR.LE.ITRMX) GO TO 304
            C
```

```
ISN 0237          92 IF(ISAVE .NE.1) GO TO 30
             C
             C    SAVE VALUES ON TAPE
             C
ISN 0239             WRITE(2) LMX,KMX,NMX,DPHI
ISN 0240             DO 95 N=1,NMX
ISN 0241             WRITE(2) ((F(L,K,N),L=1,LMX),K=1,KMX)
ISN 0242          95 CONTINUE
             C
             C    END OF TAPE WRITE
             C
ISN 0243          30 CONTINUE
ISN 0244             CALL OUTPUT(5,IBC)
ISN 0245             STOP
ISN 0246             END
```

```
LEVEL 21.7 ( DEC 72 )                                        OS/360  FORTRAN H

                COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                                   SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
                      C
  ISN 0002                  SUBROUTINE GRID
  ISN 0003                  COMMON ZT(30),XI(60),RO(10),F(30,60,10),A(10),B(10),D(10),E(10),
                          *    EE(30),FF(30),SV(30),DPHI(10),BV(10),US(30,60),UN(30,60),
                          *    DNDS(30,10,2),FAK(60),FHW(60),UTA(30),UTB(30),UTC(30),BBV(30),
                          *    BSA(30),BSB(30),BSC(30),VSA(30),VSB(30),BAV(30),LOP(30)
                          *    ,SONIC(10)
  ISN 0004                  COMMON DAF,DBF,DCF,AKLKE,RX,OX,CONST
  ISN 0005                  COMMON H,DRO,DZT,TDZ,TPB,XIC,XIB,XID,TDR,AKLK,DRO2,DZT2
  ISN 0006                  COMMON OMCU,RHUB,RTIP,R2,RXE,RXH,UA,UB,TRDZ,ERA,ERB,CKLKE,SB,
                          *    BTR,FXB,FXI,EM2,AMA,AMB,DM,DKA,DKR,DQ,DQS,TZ,TTZ,TZS,TRS,TDQ
  ISN 0007                  COMMON K,N,IDM,     KMX,LMX,NMX,KMXM1,KMXM2,LMXM1,LMXM2,NMXM1,
                          *    NMXM2,KTEO,IRX,    ITK,ITR,ITRMX,KLEP,ITPR,KUP,KDN
  ISN 0008                  COMMON ID(36)
  ISN 0009                  XIC = 0.0
  ISN 0010                  XID = OMCU
  ISN 0011                  XIB =  FXB*XID
  ISN 0012                  XII = XID + FXI*XID
  ISN 0013                  DQ = 2.0/FLOAT(KMX+1)
  ISN 0014                  TDQ = 2.*DQ
  ISN 0015                  DQS = DQ*DQ
  ISN 0016                  XIMP = (XIB + XII)/2.
  ISN 0017                  TALF = ALOG(DQ/(2.0-DQ))/( FXB*XID-XIMP)
  ISN 0018                  ALF = TALF/2.
  ISN 0019                  DO 1 K = 1,KMX
  ISN 0020                  Q = -1.0 + FLOAT(K)*DQ
  ISN 0021                  FAK(K) = ALF*(1. - Q*Q)
  ISN 0022                  BKT = DQ*(0.5+FLOAT(K))-1.
  ISN 0023                  FHW(K) = ALF*(1. - BKT*BKT)
  ISN 0024                1 XI(K) = XIMP + ALOG((1.0+Q)/(1.0-Q)) / TALF
  ISN 0025                4 DZT = TPB/FLOAT(LMXM1)
  ISN 0026                  DZT2 = DZT*DZT
  ISN 0027                  TDZ = 2.0*DZT
  ISN 0028                  DO 2 L = 1,LMX
  ISN 0029                2 ZT(L) = FLOAT(L-1)*DZT
  ISN 0030                  DRO = (RTIP-RHUB)/FLOAT(NMXM1)
  ISN 0031                  TDR = 2.*DRO
  ISN 0032                  DRO2 = DRO*DRO
  ISN 0033                  DO 3 N = 1,NMX
  ISN 0034                3 RO(N) = FLOAT(N-1)*DRO   + RHUB
  ISN 0035                  TZ = DQ/DZT
  ISN 0036                  TZS = TZ*TZ
  ISN 0037                  TTZ = DQ/TDZ
  ISN 0038                  TRS = DQS/DRO2
  ISN 0039                  DAF = (XI(KMX)-XI(KMXM1))/((XI(KMXM1)-XI(KMXM2))*(XI(KMX)
                          *    -XI(KMXM2)))
  ISN 0040                  DBF = (XI(KMX)-XI(KMXM2))/((XI(KMXM1)-XI(KMXM2))*(XI(KMXM1)
                          *    -XI(KMX)))
  ISN 0041                  DCF = (2.*XI(KMX)-XI(KMXM1)-XI(KMXM2))/((XI(KMX)-XI(KMXM2))*
                          *    (XI(KMX)-XI(KMXM1)))
  ISN 0042                  IF(IDM.EQ.2) GO TO 13
  ISN 0044                  DO 12 N = 1,NMX
  ISN 0045                  R2 = RO(N)*RO(N)
  ISN 0046                  E(N) = 1.0 + R2
  ISN 0047                  D(N) = (E(N)*E(N))/R2
  ISN 0048                  B(N) = -E(N)
```

```
ISN 0049         12 A(N) = R2
ISN 0050            GO TO 14
ISN 0051         13 DO 15 N = 1,NMX
ISN 0052            R2 = RO(N)*RO(N)
ISN 0053            E(N) = 0.0
ISN 0054            D(N) = (1.+R2)*(1.+R2)/R2
ISN 0055            B(N) = -1.-R2
ISN 0056            A(N) = R2
ISN 0057         15 CONTINUE
ISN 0058         14 CONTINUE
ISN 0059            DO 40 K = 1,KMX
ISN 0060            IF(XI(K).GT.XID) GO TO 41
ISN 0062         40 CONTINUE
ISN 0063         41 KTEO = K
ISN 0064            WRITE(6,201) XID
ISN 0065        201 FORMAT(/5X,'THE BLADES LIE BETWEEN Z = 0  AND',F9.6)
ISN 0066            WRITE(6,202)
ISN 0067        202 FORMAT(//4X,'K',7X,' Z',10X,' X/CA ',8X,'RXE', 9X,'RXH',/)
ISN 0068            DO 11 K = 1,KMX
ISN 0069            PCT = XI(K)/XID
ISN 0070            RX = 1. + (RXE-1.)*EXP(-((XI(K)-0.5*XID)/XID)**2)
ISN 0071            IF(ITPR.EQ.0) RX = RXE
ISN 0073            WRITE(6,203) K,XI(K),PCT,RX,RXH
ISN 0074         11 CONTINUE
ISN 0075        203 FORMAT(3X,I2,4X,F9.6,5X,F7.4,5X,F7.4,5X,F7.4)
ISN 0076            WRITE(6,204)
ISN 0077        204 FORMAT(//4X,'L     ZETA',/)
ISN 0078            DO 50 L = 1,LMX
ISN 0079            WRITE(6,205) L,ZT(L)
ISN 0080         50 CONTINUE
ISN 0081        205 FORMAT(3X,I2,F9.4,8X,F9.6)
ISN 0082            KHW = INT(FLOAT(KMX)/2.)
ISN 0083            WRITE(6,208) KHW
ISN 0084        208 FORMAT(//4X,'OPTIMUM SHOCK CAPTURING OCCURS WHEN THE GRID-SIZE',
                   *    ' RATIO RHO*(DELTA ZETA)/(DELTA Z) = (1+RHO**2)/RHO',
                   *    /4X,'THE TABLE BELOW LISTS THESE OPTIMUM VALUES, COMPARED ',
                   *    'TO THE VALUES ACTUALLY USED AT K =',I2,)
ISN 0085            WRITE(6,206) KHW
ISN 0086        206 FORMAT(//4X,'N     RHO',7X,'US/WO CRIT',  5X,' ARCTAN(RHO),',
                   *    6X,'M REL',16X,'GRID-SIZE RATIO',
                   *    /36X,'DEGREES',
                   *    26X,'OPTIMUM     ACTUAL(AT K = ',I2,')',/)
ISN 0087            DO 10 N = 1,NMX
ISN 0088            BTR = 1.0 - EM2*(1.0+RO(N)*RO(N))
ISN 0089            IF(SB.EQ.0.) GO TO 51
ISN 0091            USCRIT = BTR/(SB*(1.+RO(N)*RO(N)))
ISN 0092         51 PSI = 57.29578*ATAN(RO(N))
ISN 0093            SONIC(N) = USCRIT
ISN 0094            REL = SQRT(EM2*(1.+A(N)))
ISN 0095            OPT = (1.+RO(N)*RO(N))/RO(N)
ISN 0096            ACT = RO(N)*DZT*FHW(KHW)/DQ
ISN 0097            WRITE(6,207) N,RO(N),USCRIT,PSI,REL,OPT,ACT
ISN 0098         10 CONTINUE
ISN 0099        207 FORMAT(3X,I2,F9.4,1PE15.4,6X,0PF8.3,9X,F6.4,11X,F6.4,9X,F6.4)
ISN 0100            RETURN
ISN 0101            END
```

LEVEL 21.7 (DEC 72)   OS/360 FORTRAN H

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          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
               C
 ISN 0002            SUBROUTINE BVALO(I)
 ISN 0003            COMMON ZT(30),XI(60),RO(10),F(30,60,10),A(10),B(10),D(10),E(10),
                   *    EE(30),FF(30),SV(30),DPHI(10),BV(10),US(30,60),UN(30,60),
                   *    DNDS(30,10,2),FAK(60),FHW(60),UTA(30),UTB(30),UTC(30),BBV(30),
                   *    BSA(30),BSB(30),BSC(30),VSA(30),VSB(30),BAV(30),LOP(30)
                   *    ,SONIC(10)
 ISN 0004            COMMON DAF,DBF,DCF,AKLKE,RX,OX,CONST
 ISN 0005            COMMON H,DRO,DZT,TDZ,TPB,XIC,XIB,XID,TDR,AKLK,DRO2,DZT2
 ISN 0006            COMMON OMCU,RHUB,RTIP,R2,RXE,RXH,UA,UB,TRDZ,ERA,ERB,CKLKE,SB,
                   *    BTR,FXB,FXI,EM2,AMA,AMB,DM,DKA,DKR,DQ,DQS,TZ,TTZ,TZS,TRS,TDQ
 ISN 0007            COMMON K,N,IDM,     KMX,LMX,NMX,KMXM1,KMXM2,LMXM1,LMXM2,NMXM1,
                   *    NMXM2,KTEO,IRX,    ITK,ITR,ITRMX,KLEP,ITPR,KUP,KDN
 ISN 0008            COMMON ID(36)
 ISN 0009            IF(XI(K).LT.XIC) GO TO 1
 ISN 0011            IF(XI(K).LT.XID) GO TO 2
 ISN 0013            GO TO 3
 ISN 0014          1 EE(1) = 0.
 ISN 0015            FF(1) = F(LMX,K,N)
 ISN 0016            GO TO 10
 ISN 0017          2 GO TO (100,200),I
 ISN 0018        200 EE(1) = 0.
 ISN 0019            KB = K - KLEP
 ISN 0020            FF(1) = F(LMX,K,N) + DNDS(KB,N,1)
 ISN 0021            GO TO 10
 ISN 0022        100 SLP = DNDS(K-KLEP,N,1)
 ISN 0023            AE = -BSA(1)*2.*FAK(K)/RX + 2.*BSB(1)*FHW(K-1) -3.5*AKLKE
                   *    - BSC(1)*FHW(K-1) +4.*DKA
 ISN 0024            BE = -8.*DKA + 4.*AKLKE
 ISN 0025            CE = BSA(1)*(FHW(K)*F(1,K+1,N)-2.*FAK(K)*OX*UTA(1)+FHW(K-1)
                   *    *F(1,K-1,N)) + BSB(1)*(-FHW(K-1)*(UTA(1)+F(1,K-1,N))
                   *    -FHW(K-2)*(F(1,K-1,N)-UTC(1))) + BSC(1)*(FHW(K)*(F(1,K+1,N)
                   *    -UTA(1))+FHW(K-1)*F(1,K-1,N))
                   *    -DKA*(UTC(3)-UTA(3)-8.*UTB(2)+8.*UTB(1)-UTC(1)-3.*UTA(1))
                   *    +AKLKE*(-.5*UTA(3)-3.*RO(N)*DZT*SLP-A(N)*DZT*UB*(UTC(1)
                   *    -6.*UTB(1)+3.*UTA(1)+2.*F(1,K+1,N))*FAK(K))
                   *    +DKR*(ERA*F(1,K,N+1)+ERB*F(1,K,N-1)-2.*F(1,K,N))
 ISN 0026            EE(1) = -BE/AE
 ISN 0027         12 FF(1) = -CE/AE
 ISN 0028            GO TO 10
 ISN 0029          3 EE(1) = 0.0
 ISN 0030            Z = F(LMX,K,N)
 ISN 0031          6 FF(1) = Z + DPHI(N)
 ISN 0032         10 RETURN
 ISN 0033            END
```

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LEVEL 21.7 ( DEC 72 )                              OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
                 C
  ISN 0002             SUBROUTINE BVAL1(I)
  ISN 0003             COMMON ZT(30),XI(60),RO(10),F(30,60,10),A(10),B(10),D(10),E(10),
                     *    EE(30),FF(30),SV(30),DPHI(10),BV(10),US(30,60),UN(30,60),
                     *    DNDS(30,10,2),FAK(60),FHW(60),UTA(30),UTB(30),UTC(30),BBV(30),
                     *    BSA(30),BSB(30),BSC(30),VSA(30),VSB(30),BAV(30),LOP(30)
                     *    ,SONIC(10)
  ISN 0004             COMMON DAF,DBF,DCF,AKLKE,RX,OX,CONST
  ISN 0005             COMMON H,DRO,DZT,TDZ,TPB,XIC,XIB,XID,TDR,AKLK,DRO2,DZT2
  ISN 0006             COMMON OMCU,RHUB,RTIP,R2,RXE,RXH,UA,UB,TRDZ,ERA,ERB,CKLKE,SB,
                     *    BTR,FXB,FXI,EM2,AMA,AMB,DM,DKA,DKR,DQ,DQS,TZ,TTZ,TZS,TRS,TDQ
  ISN 0007             COMMON K,N,IDM,     KMX,LMX,NMX,KMXM1,KMXM2,LMXM1,LMXM2,NMXM1,
                     *    NMXM2,KTEO,IRX,    ITK,ITR,ITRMX,KLEP,ITPR,KUP,KDN
  ISN 0008             COMMON ID(36)
  ISN 0009             FPA = F(2,K-1,N)
  ISN 0010             FPB = F(2, K ,N)
  ISN 0011             FPC = F(2,K+1,N)
  ISN 0012             DKL = 0.
  ISN 0013             IF(XI(K).LT.XIC) GO TO 1
  ISN 0015             IF(XI(K).LT.XID) GO TO 2
  ISN 0017             FPA = FPA - DPHI(N)
  ISN 0018             FPB = FPB - DPHI(N)
  ISN 0019             FPC = FPC - DPHI(N)
  ISN 0020             IF(IDM.EQ.2) GO TO 1
  ISN 0022             DKL = (1.+RO(N)*RO(N))*
                     *       0.5*TRS*(ERA*DPHI(N+1) + ERB*DPHI(N-1) -2.*DPHI(N))/FAK(K)
  ISN 0023           1 L = LMX
  ISN 0024             BKL = -2.*AKLKE - 2.*FAK(K)*BSA(L)/RX + 2.*FHW(K-1)*BSB(L)
                     *    - BSC(L)*FHW(K-1)
  ISN 0025             DKL = DKL - BSA(L)*(FHW(K)*F(L,K+1,N)-2.*FAK(K)*OX*UTA(L)
                     *    + FHW(K-1)*F(L,K-1,N)) + BSB(L)*(FHW(K-1)*(UTA(L)+F(L,K-1,N))
                     *    + FHW(K-2)*(F(L,K-1,N)-UTC(L))) - BSC(L)*(FHW(K)*(F(L,K+1,N)
                     *    -UTA(L)) + FHW(K-1)*F(L,K-1,N))
                     *    +2.*DKA*(-FPA+F(L,K-1,N)+FPB-2.*F(L,K,N)+F(L-1,K,N)
                     *    +F(L,K+1,N)-F(L-1,K+1,N))
                     *    - DKR*(ERA*F(L,K,N+1)+ERB*F(L,K,N-1)-2.*UTA(L))
  ISN 0026          61 AKL = AKLKE
  ISN 0027             CKL = CKLKE
  ISN 0028          25 SV(LMX) = (DKL-AKL*FPB      -CKL*FF(LMXM1))/(BKL+CKL*EE(LMXM1))
  ISN 0029             GO TO 10
  ISN 0030           2 CONTINUE
  ISN 0031             GO TO (100,200),I
  ISN 0032         200 SLP = DNDS(K-KLEP,N,2)
  ISN 0033             AE = -2.*FAK(K)*BSA(LMX)/RX + 2.*BSB(LMX)*FHW(K-1) - 7.*AKLKE/2.
                     *    -BSC(LMX)*FHW(K-1) + DKA
  ISN 0034             BE = -8.*DKA + 4.*AKLKE
  ISN 0035             CE = BSA(LMX)*(FHW(K)*F(LMX,K+1,N)-2.*FAK(K)*UTA(LMX)*OX
                     *    + FHW(K-1)*F(LMX,K-1,N)) + BSB(LMX)*(-FHW(K-1)*(UTA(LMX)
                     *    + F(LMX,K-1,N)) - FHW(K-2)*(F(LMX,K-1,N)-UTC(LMX)))
                     *+BSC(LMX)*(FHW(K)*(F(LMX,K+1,N)-UTA(LMX))+FHW(K-1)*F(LMX,K-1,N))
                     *    -DKA*(F(LMXM2,K+2,N)-UTA(LMXM2)-8.*F(LMXM1,K+1,N)+4.*(
                     *    F(LMX,K+1,N)-UTB(LMX)) + UTC(LMX))
                     *    +AKLKE*(-F(LMXM2,K,N)-6.*DZT*SLP           -11.*UTA(1)+18.*
                     *    UTA(2)-9.*UTA(3)+2.*UTA(4))/2.
                     *    +DKR*(ERA*F(LMX,K,N+1)+ERB*F(LMX,K,N-1)-2.*F(LMX,K,N))
  ISN 0036             GO TO 11
```

```
ISN 0037        100 SLP = DNDS(K-KLEP,N,2)
ISN 0038              L = LMX
ISN 0039              AE = -2.*FAK(K)*BSA(LMX)/RX + 2.*BSB(LMX)*FHW(K-1)
                   *     -BSC(LMX)*FHW(K-1)   + DKA  - 3.5*AKLKE
ISN 0040              BE = 4.*AKLKE    -8.*DKA
ISN 0041              CE = BSA(LMX)*(FHW(K)*F(LMX,K+1,N)-2.*FAK(K)*UTA(LMX)*OX
                   *     +FHW(K-1)*F(LMX,K-1,N))+BSB(LMX)*(-FHW(K-1)*(UTA(LMX)+F(LMX,
                   *     K-1,N))-FHW(K-2)*(F(LMX,K-1,N)-UTC(LMX)))+BSC(LMX)*(FHW(K)*
                   *     (F(LMX,K+1,N)-UTA(LMX))+FHW(K-1)*F(LMX,K-1,N))
                   *     -DKA*(F(LMXM2,K+2,N)-UTA(LMXM2)-8.*F(LMXM1,K+1,N)+4.*F(LMX,
                   *     K+1,N)-4.*UTB(LMX)+UTC(LMX))+AKLKE*(-0.5*UTA(LMXM2)+3.*RO(N)
                   *     *DZT*SLP+A(N)*DZT*UB*(UTC(LMX)-6.*UTB(LMX)+3.*UTA(LMX)+2.*
                   *     F(LMX,K+1,N))*FAK(K))
                   *     +DKR*(ERA*F(LMX,K,N+1)+ERB*F(LMX,K,N-1)-2.*F(LMX,K,N))
ISN 0042         11 SV(LMX) = -(CE+BE*FF(LMXM1))/(AE+BE*EE(LMXM1))
ISN 0043         10 RETURN
ISN 0044            END
```

```
LEVEL 21.7 ( DEC 72 )                              OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
                    C
   ISN 0002                 SUBROUTINE OUTPUT(IOP,IBC)
   ISN 0003                 COMMON ZT(30),XI(60),RO(10),F(30,60,10),A(10),B(10),D(10),E(10),
                           *   EE(30),FF(30),SV(30),DPHI(10),BV(10),US(30,60),UN(30,60),
                           *   DNDS(30,10,2),FAK(60),FHW(60),UTA(30),UTB(30),UTC(30),BBV(30),
                           *   BSA(30),BSB(30),BSC(30),VSA(30),VSB(30),BAV(30),LOP(30)
                           *   ,SONIC(10)
   ISN 0004                 COMMON DAF,DBF,DCF,AKLKE,RX,OX,CONST
   ISN 0005                 COMMON H,DRO,DZT,TDZ,TPB,XIC,XIB,XID,TDR,AKLK,DRO2,DZT2
   ISN 0006                 COMMON OMCU,RHUB,RTIP,R2,RXE,RXH,UA,UB,TRDZ,ERA,ERB,CKLKE,SB,
                           *   BTR,FXB,FXI,EM2,AMA,AMB,DM,DKA,DKR,DQ,DQS,TZ,TTZ,TZS,TRS,TDQ
   ISN 0007                 COMMON K,N,IDM,     KMX,LMX,NMX,KMXM1,KMXM2,LMXM1,LMXM2,NMXM1,
                           *   NMXM2,KTEO,IRX,    ITK,ITR,ITRMX,KLEP,ITPR,KUP,KDN
   ISN 0008                 COMMON ID(36)
   ISN 0009                 DIMENSION UR(30,60)
   ISN 0010                 IF(IOP.EQ.2) GO TO 2
   ISN 0012                 IF(IOP.EQ.3) GO TO 20
   ISN 0014                 IF(IOP.EQ.4) GO TO 20
   ISN 0016                 IF(IOP.EQ.5) GO TO 90
   ISN 0018                 IF(IOP.EQ.6) GO TO 96
                    C
                    C    IOP = 1 GIVES VALUES OF THE POTENTIAL AND ALL VELOCITY COMPONENTS
                    C    AT ALL GRID POINTS.
                    C    IOP = 2 GIVES ONLY VELOCITY COMPONENTS, AND ONLY AT ZETA = 0. AND 2.*PI/B
                    C    IOP = 3 IS THE SAME AS IOP = 1, BUT WITH VALUES OF THE POTENTIAL
                    C    OMITTED.
                    C    IOP = 4 GIVES MACH NUMBER CONTOURS.
                    C    IOP = 5 CALCULATES PERFORMANCE DATA
                    C    IOP = 6 GIVES THE LOCAL MACH NUMBER AT ALL POINTS
                    C    IOP = 7 PRINTS ONLY THE POTENTIAL (AT ALL POINTS)
                    C
                    C    PRINT PHI
                    C
   ISN 0020                 LA = 1
   ISN 0021                 LB = 10
   ISN 0022                 WRITE(6,250) ITR,ITK,N,RO(N)
   ISN 0023             250 FORMAT(1H1,
                           *          //3X,'VALUES OF THE POTENTIAL AFTER ITR = ',I4,
                           *    ', ITK = ',I4,
                           *    '  AT RHO(',I2,') = ',F7.4)
   ISN 0024              10 WRITE(6,200) (LOP(L),L=LA,LB)
   ISN 0025             200 FORMAT(/4X,'K    L=',I3,9I12,//)
   ISN 0026                 DO 80 K = 1,KMX
   ISN 0027                 WRITE(6,251)  K,(F(L,K,N),L=LA,LB)
   ISN 0028              80 CONTINUE
   ISN 0029             251 FORMAT(I5,1P10E12.3)
   ISN 0030                 LA = LA + 10
   ISN 0031                 IF(LA.GT.LMX) GO TO 20
   ISN 0033                 LB = LB + 10
   ISN 0034                 IF(LB.GT.LMX) LB=LMX
   ISN 0036                 GO TO 10
   ISN 0037              20 CONTINUE
   ISN 0038                 IF(IOP.EQ.7) RETURN
                    C
                    C    CALCULATE US AND UN
                    C
```

```
ISN 0040          DO 60 K = 2,KMXM1
ISN 0041          DO 61 L = 2,LMXM1
ISN 0042          US(L,K) = UB*FAK(K)*(F(L,K+1,N) - F(L,K-1,N))
ISN 0043       61 UN(L,K) = ((F(L+1,K-1,N)-F(L,K-1,N))+(F(L,K+1,N)-F(L-1,K+1,N)))
                 *    /TRDZ - UA*US(L,K)
ISN 0044       60 CONTINUE
ISN 0045          DO 63 K = 2,KMXM1
ISN 0046          US(1,K) = UB*FAK(K)*(F(1,K+1,N)-F(1,K-1,N))
ISN 0047          US(LMX,K) = UB*FAK(K)*(F(LMX,K+1,N)-F(LMX,K-1,N))
ISN 0048          IF(K.LE.KLEP) GO TO 62
ISN 0050          IF(K.GE.KTEO) GO TO 62
ISN 0052          UN(1,K) = (-3.*F(1,K,N)+4.*F(2,K-1,N)-F(3,K-2,N)
                 *   +F(1,K+1,N)-F(1,K-1,N))/TRDZ - UA*US(1,K)
ISN 0053          UN(LMX,K) = (3.*F(LMX,K,N)-4.*F(LMXM1,K+1,N)+F(LMXM2,K+2,N)
                 *   + F(LMX,K+1,N)-F(LMX,K-1,N))/TRDZ - UA*US(LMX,K)
ISN 0054          GO TO 63
ISN 0055       62 FZTA = (F(2,K-1,N)-F(1,K-1,N)+F(LMX,K+1,N)-F(LMXM1,K+1,N))/TRDZ
ISN 0056          UN(1,K) = FZTA - UA*US(1,K)
ISN 0057          UN(LMX,K) = FZTA - UA*US(LMX,K)
ISN 0058       63 CONTINUE
             C
             C    FIND MACH NUMBER CONTOURS
             C
ISN 0059          IF(ICP.NE.4) GO TO 11
ISN 0061          TGTM = AMA
ISN 0062          RELSQ = EM2*(1.+A(N))
ISN 0063          GMAP = SB/EM2
ISN 0064          REL = SQRT(RELSQ)
ISN 0065          CF = GMAP/2.
ISN 0066      504 CONTINUE
ISN 0067          CALL CLEAR (UR(1,1),UR(30,60))
ISN 0068          CALL CLEAR (UN(1,1),UN(30,60))
             C
             C    THE FOLLOWING FORMULA WAS USED IN CALCULATING THE MACH NUMBER
             C    CONTOURS IN AIAA PAPER 77-199
             C      TGT = ((TGTM*TGTM/RELSQ)-1.)/GMAP
             C
             C    THE FOLLOWING IS THE LINEARIZED FORMULA
             C
ISN 0069          TGT =-(1.-TGTM/REL)/CF
             C
ISN 0070          KWMX = 0
ISN 0071          DO 500 L = 1,LMX
ISN 0072          SGN = (US(L,KUP) - TGT)/ABS(US(L,KUP)-TGT)
ISN 0073          KW = 1
ISN 0074          DO 501 K = KUP,KDN
ISN 0075          IF(SGN.GT.0.) GO TO 502
ISN 0077          IF((US(L,K)-TGT).GT.0.) GO TO 503
ISN 0079          GO TO 501
ISN 0080      502 IF((US(L,K)-TGT).LT.0.) GO TO 503
ISN 0082          GO TO 501
ISN 0083      503 X2 = XI(K)
ISN 0084          X1 = XI(K-1)
ISN 0085          Y2 = US(L,K)
ISN 0086          Y1 = US(L,K-1)
ISN 0087          Z = X1 + (X2-X1)*(TGT-Y1)/(Y2-Y1)
ISN 0088          UR(L,KW) = Z/XTD
ISN 0089          UN(L,KW) = RO(N)*(ZT(L)+Z)/OMCU
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```
ISN 0090          IF(KW.GT.KWMX) KWMX = KW
ISN 0092          KW = KW + 1
ISN 0093          SGN = -SGN
ISN 0094      501 CONTINUE
ISN 0095      500 CONTINUE
ISN 0096          IF(KWMX.EQ.0) GO TO 510
ISN 0098          WRITE(6,205) N,RO(N),TGTM
ISN 0099      205 FORMAT(/5X,'FOR RO(',I2,') = ',F7.4,' THE LOCAL MACH NUMBER',
         *     ' EQUALS',F7.4,' AT THE FOLLOWING VALUES OF X/CA AND ',
         *     'R*THETA/CA',/)
ISN 0100          LA = 1
ISN 0101          LB = 20
ISN 0102          IF(LMX.LT.20) LB=LMX
ISN 0104      511 WRITE(6,230) (LOP(L),L=LA,LB)
ISN 0105      230 FORMAT(/1X,'KW L=',I2,19I6,//)
ISN 0106      231 FORMAT(1X,I2,20F6.2)
ISN 0107          DO 512 KW = 1,KWMX
ISN 0108          WRITE(6,231) KW,(UR(L,KW),L=LA,LB)
ISN 0109          WRITE(6,206) (UN(L,KW),L=LA,LB)
ISN 0110          WRITE(6,207)
ISN 0111      512 CONTINUE
ISN 0112      520 CONTINUE
ISN 0113      206 FORMAT(3X,20F6.2)
ISN 0114      207 FORMAT(1X)
ISN 0115          LA = LA + 10
ISN 0116          IF(LA.GT.LMX) GO TO 510
ISN 0118          LB = LB + 10
ISN 0119          IF(LB.GT.LMX) LB=LMX
ISN 0121          GO TO 511
ISN 0122      510 TGTM = TGTM + DM
ISN 0123          IF(TGTM.LE.AMB) GO TO 504
ISN 0125          RETURN
            C
            C     PRINT US
            C
ISN 0126       11 LA = 1
ISN 0127          LB = 10
ISN 0128          WRITE(6,253) ITR,ITK,N,RO(N),SONIC(N)
ISN 0129      253 FORMAT(///3X,'VALUES OF US/WO AFTER ITR = ',I4,', ITK = ',I4,
         *     '   AT RHO(',I2,') = ',F7.4,10X,'US/WO CRIT = ',1PE12.3)
ISN 0130       21 WRITE(6,200) (LOP(I),I=LA,LB)
ISN 0131          DO 81 K = 2,KMXM1
ISN 0132          WRITE(6,251)  K,(US(L,K),L=LA,LB)
ISN 0133       81 CONTINUE
ISN 0134          LA = LA + 10
ISN 0135          IF(LA.GT.LMX) GO TO 40
ISN 0137          LB = LB + 10
ISN 0138          IF(LB.GT.LMX) LB = LMX
ISN 0140          GO TO 21
            C
            C     PRINT UN
            C
ISN 0141       40 WRITE(6,254) ITR,ITK,N,RO(N)
ISN 0142          LA = 1
ISN 0143          LB = 10
ISN 0144      254 FORMAT(///3X,'VALUES OF UN/WO AFTER ITR = ',I4,', ITK = ',I4,
         *     '   AT RHO(',I2,') = ',F7.4)
ISN 0145       41 WRITE(6,200) (LOP(I),I=LA,LB)
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ISN 0146          DO 82 K = 2,KMXM1
ISN 0147          WRITE(6,251)  K,(UN(L,K),L=LA,LB)
ISN 0148       82 CONTINUE
ISN 0149          LA = LA + 10
ISN 0150          IF(LA.GT.LMX) GO TO 31
ISN 0152          LB = LB + 10
ISN 0153          IF(LB.GT.LMX) LB = LMX
ISN 0155          GO TO 41
ISN 0156       31 IF(IDM.EQ.2) WRITE(6,210)
ISN 0158      210 FORMAT(//10X,'NOTE: THE FOLLOWING VALUES OF V RADIAL ARE BASED',
                *    ' ON THE 2D STRIP-THEORY APPROXIMATION')
            C
            C   CALCULATE V RADIAL
            C
ISN 0159          IF(N.EQ.1) GO TO 52
ISN 0161          IF(N.EQ.NMX) GO TO 52
ISN 0163          DO 50 K = 2,KMXM1
ISN 0164          DO 51 L = 1,LMX
ISN 0165          UR(L,K) = (F(L,K,N+1) - F(L,K,N-1))/TDR
ISN 0166       51 CONTINUE
ISN 0167       50 CONTINUE
ISN 0168          GO TO 53
ISN 0169       52 DO 54 K = 2,KMXM1
ISN 0170          DO 55 L = 1,LMX
ISN 0171          UR(L,K) = 0.
ISN 0172       55 CONTINUE
ISN 0173       54 CONTINUE
            C
            C   PRINT V RADIAL
            C
ISN 0174       53 WRITE(6,255) ITR,ITK,N,RO(N)
ISN 0175      255 FORMAT(///3X,'VALUES OF V RADIAL/U INFINITY',
                *                                'AFTER ITR = ',I4,', ITK = ',I4,
                *    '  AT RHO(',I2,') = ',F7.4)
ISN 0176          LA = 1
ISN 0177          LB = 10
ISN 0178       56 WRITE(6,200) (LOP(I),I=LA,LB)
ISN 0179          DO 83 K = 2,KMXM1
ISN 0180          WRITE(6,251)  K,(UR(L,K),L=LA,LB)
ISN 0181       83 CONTINUE
ISN 0182          LA = LA + 10
ISN 0183          IF(LA.GT.LMX) GO TO 30
ISN 0185          LB = LB + 10
ISN 0186          IF(LB.GT.LMX) LB = LMX
ISN 0188          GO TO 56
            C
            C   CALCULATE AND PRINT VELOCITIES ON ZETA = 0 AND ZETA1
            C
ISN 0189        2 WRITE(6,201) ITR,ITK,N,RO(N),SONIC(N)
ISN 0190      201 FORMAT(//4X,'SURFACE VELOCITIES AFTER ITR = ',I4,', ITK = ',I4,
                *    '  AT RHO(',I2,') = ',F7.4,10X,'US/WO CRIT = ',1PE12.3,
                *    /4X,'K',4X,'US(1,K)',4X,'US(LMX,K)',4X,'UN(1,K)',4X,
                *    'UN(LMX,K)',4X,'UR(1,K)',4X,'UR(LMX,K)',/)
ISN 0191          DO 65 K = 2,KMXM1
ISN 0192          US(1,K) = UB*FAK(K)*(F(1,K+1,N)-F(1,K-1,N))
ISN 0193          US(LMX,K) = UB*FAK(K)*(F(LMX,K+1,N)-F(LMX,K-1,N))
ISN 0194          IF(K.LE.KLEP) GO TO 64
ISN 0196          IF(K.GE.KTEO) GO TO 64
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ISN 0198          UN(1,K) = (-3.*F(1,K,N)+4.*F(2,K-1,N)-F(3,K-2,N)
                 *    +F(1,K+1,N)-F(1,K-1,N))/TRDZ - UA*US(1,K)
ISN 0199          UN(LMX,K) = (3.*F(LMX,K,N)-4.*F(LMXM1,K+1,N)+F(LMXM2,K+2,N)
                 *    + F(LMX,K+1,N)-F(LMX,K-1,N))/TRDZ - UA*US(LMX,K)
ISN 0200          GO TO 66
ISN 0201       64 FZTA = (F(2,K-1,N)-F(1,K-1,N)+F(LMX,K+1,N)-F(LMXM1,K+1,N))/TRDZ
ISN 0202          UN(1,K) = FZTA - UA*US(1,K)
ISN 0203          UN(LMX,K) = FZTA - UA*US(LMX,K)
ISN 0204       66 IF(N.EQ.1) GO TO 70
ISN 0206          IF(N.EQ.NMX) GO TO 70
ISN 0208          UR(1,K) = (F(1,K,N+1) - F(1,K,N-1))/TDR
ISN 0209          UR(LMX,K) = (F(LMX,K,N+1) - F(LMX,K,N-1))/TDR
ISN 0210          GO TO 72
ISN 0211       70 UR(1,K) = 0.0
ISN 0212          UR(LMX,K) = 0.0
ISN 0213       72 WRITE(6,251) K,US(1,K),US(LMX,K),UN(1,K),UN(LMX,K),
                 *    UR(1,K),UR(LMX,K)
ISN 0214       65 CONTINUE
ISN 0215       30 CONTINUE
ISN 0216          K1 = KTEO
ISN 0217          K2 = KTEO - 1
ISN 0218          K3 = KTEO - 2
ISN 0219          X1 = XI(K1)
ISN 0220          X2 = XI(K2)
ISN 0221          X3 = XI(K3)
ISN 0222          Y1 = US(1,K1)
ISN 0223          Y2 = US(1,K2)
ISN 0224          Y3 = US(1,K3)
ISN 0225          X = XID
ISN 0226          Z = (X-X2)*(X-X3)*Y1/((X1-X2)*(X1-X3))
                 *    +(X-X1)*(X-X3)*Y2/((X2-X1)*(X2-X3))
                 *    +(X-X1)*(X-X2)*Y3/((X3-X1)*(X3-X2))
ISN 0227          U1 = Z
ISN 0228          Y1 = US(LMX,K1)
ISN 0229          Y2 = US(LMX,K2)
ISN 0230          Y3 = US(LMX,K3)
ISN 0231          Z = (X-X2)*(X-X3)*Y1/((X1-X2)*(X1-X3))
                 *    +(X-X1)*(X-X3)*Y2/((X2-X1)*(X2-X3))
                 *    +(X-X1)*(X-X2)*Y3/((X3-X1)*(X3-X2))
ISN 0232          U2 = Z
ISN 0233          DU = U1 - U2
ISN 0234          WRITE(6,202) U1,U2,DU,N,DPHI(N)
ISN 0235      202 FORMAT( /5X,'TRAILING-EDGE VELOCITIES ARE US(1,TE) = ',1PE10.3,
                 *    '  US(LMX,TE) = ',E10.3,'  DELTA US = ',E10.3,
                 *    '  DPHI(',I2,') = ',E10.3)
ISN 0236          IF(IBC.NE.2) RETURN
                C
                C   CALCULATE THE BLADE SHAPE
                C
ISN 0238          IF(IOP.NE.6) GO TO 22
ISN 0240          K1 = KTEO
ISN 0241          K2 = KTEO - 1
ISN 0242          K3 = KTEO - 2
ISN 0243          X1 = XI(K1)
ISN 0244          X2 = XI(K2)
ISN 0245          X3 = XI(K3)
ISN 0246          K0 = KLEP - 3
ISN 0247          K4 = KTEO + 3
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ISN 0248          DO 23 K = K0,K4
ISN 0249          US(1,K) = UB*FAK(K)*(F(1,K+1,N)-F(1,K-1,N))
ISN 0250          US(LMX,K) = UB*FAK(K)*(F(LMX,K+1,N)-F(LMX,K-1,N))
ISN 0251          IF(K.LE.KLEP) GO TO 24
ISN 0253          IF(K.GE.KTEO) GO TO 24
ISN 0255          UN(1,K) = (-3.*F(1,K,N)+4.*F(2,K-1,N)-F(3,K-2,N)
                 *   +F(1,K+1,N)-F(1,K-1,N))/TRDZ - UA*US(1,K)
ISN 0256          UN(LMX,K) = (3.*F(LMX,K,N)-4.*F(LMXM1,K+1,N)+F(LMXM2,K+2,N)
                 *   + F(LMX,K+1,N)-F(LMX,K-1,N))/TRDZ - UA*US(LMX,K)
ISN 0257          GO TO 23
ISN 0258       24 FZTA = (F(2,K-1,N)-F(1,K-1,N)+F(LMX,K+1,N)-F(LMXM1,K+1,N))/TRDZ
ISN 0259          UN(1,K) = FZTA - UA*US(1,K)
ISN 0260          UN(LMX,K) = FZTA - UA*US(LMX,K)
ISN 0261       23 CONTINUE
ISN 0262       22 ORT = SQRT(1.+RO(N)*RO(N))/OMCU
ISN 0263          Y1 = UN(1,K1)
ISN 0264          Y2 = UN(1,K2)
ISN 0265          Y3 = UN(1,K3)
ISN 0266          X = XID
ISN 0267          Z = (X-X2)*(X-X3)*Y1/((X1-X2)*(X1-X3))
                 *   +(X-X1)*(X-X3)*Y2/((X2-X1)*(X2-X3))
                 *   +(X-X1)*(X-X2)*Y3/((X3-X1)*(X3-X2))
ISN 0268          UTEU = Z
ISN 0269          Y1 = UN(LMX,K1)
ISN 0270          Y2 = UN(LMX,K2)
ISN 0271          Y3 = UN(LMX,K3)
ISN 0272          Z = (X-X2)*(X-X3)*Y1/((X1-X2)*(X1-X3))
                 *   +(X-X1)*(X-X3)*Y2/((X2-X1)*(X2-X3))
                 *   +(X-X1)*(X-X2)*Y3/((X3-X1)*(X3-X2))
ISN 0273          UTEL = Z
ISN 0274          K1 = KLEP + 1
ISN 0275          K2 = KLEP + 2
ISN 0276          K3 = KLEP + 3
ISN 0277          Y1 = UN(1,K1)
ISN 0278          Y2 = UN(1,K2)
ISN 0279          Y3 = UN(1,K3)
ISN 0280          X1 = XI(K1)
ISN 0281          X2 = XI(K2)
ISN 0282          X3 = XI(K3)
ISN 0283          X = 0.
ISN 0284          Z = (X-X2)*(X-X3)*Y1/((X1-X2)*(X1-X3))
                 *   +(X-X1)*(X-X3)*Y2/((X2-X1)*(X2-X3))
                 *   +(X-X1)*(X-X2)*Y3/((X3-X1)*(X3-X2))
ISN 0285          ULEU = Z
ISN 0286          Y1 = UN(LMX,K1)
ISN 0287          Y2 = UN(LMX,K2)
ISN 0288          Y3 = UN(LMX,K3)
ISN 0289          Z = (X-X2)*(X-X3)*Y1/((X1-X2)*(X1-X3))
                 *   +(X-X1)*(X-X3)*Y2/((X2-X1)*(X2-X3))
                 *   +(X-X1)*(X-X2)*Y3/((X3-X1)*(X3-X2))
ISN 0290          ULEL = Z
ISN 0291          US(1,K1) = 0.5*XI(K1)*(UN(1,K1)+ULEU)
ISN 0292          US(LMX,K1) = 0.5*XI(K1)*(UN(LMX,K1)+ULEL)
ISN 0293          KA = KLEP+2
ISN 0294          KB = KTEO-1
ISN 0295          DO 75 K = KA,KB
ISN 0296          US(1,K) = US(1,K-1)+0.5*(XI(K)-XI(K-1))*(UN(1,K)+UN(1,K-1))
ISN 0297          US(LMX,K) = US(LMX,K-1)+0.5*(XI(K)-XI(K-1))*
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              *    (UN(LMX,K)+UN(LMX,K-1))
ISN 0298       75 CONTINUE
                  US(1,KTEO) = US(1,KTEO-1)+0.5*(XID-XI(KTEO-1))*
              *    (UTEU+UN(1,KTEO-1))
ISN 0300          US(LMX,KTEO) = US(LMX,KTEO-1)+0.5*(XID-XI(KTEO-1))*
              *    (UTEL+UN(LMX,KTEO-1))
ISN 0301          WRITE(6,203)
ISN 0302      203 FORMAT(//10X,'BLADE SHAPE',/ 8X,'XI(K)',8X,'PCT',6X,'NU/CA',5X,
              *    'NL/CA',5X,'H/CA',6X,'T/CA',6X,'NU/C',6X,'NL/C',6X,
              *    'H/C',7X,'T/C')
ISN 0303          K1 = KLEP + 1
ISN 0304          OPR = ORT*OMCU
ISN 0305          DO 76 K = K1,KTEO
ISN 0306          US(1,K) = ORT*US(1,K)
ISN 0307          US(LMX,K) = ORT*US(LMX,K)
ISN 0308          HCA = (US(1,K) + US(LMX,K))/2.
ISN 0309          TCA = US(1,K) - US(LMX,K)
ISN 0310          PCT = XI(K)/XID
ISN 0311          OU = US(1,K)/OPR
ISN 0312          OL = US(LMX,K)/OPR
ISN 0313          HBC = HCA/OPR
ISN 0314          TBC = TCA/OPR
ISN 0315          IF(K.EQ.KTEO) GO TO 76
ISN 0317          WRITE(6,204) XI(K),PCT,US(1,K),US(LMX,K),HCA,TCA,OU,OL,HBC,TBC
ISN 0318       76 CONTINUE
ISN 0319          PCT = 1.
ISN 0320          WRITE(6,204) XID,PCT,US(1,KTEO),US(LMX,KTEO),HCA,TCA,
              *    OU,OL,HBC,TBC
ISN 0321      204 FORMAT(1PE15.3,0P9F10.4)
ISN 0322          RETURN
            C
            C     CALCULATE PERFORMANCE DATA
            C
ISN 0323       90 CONTINUE
ISN 0324          WRITE(6,220)
ISN 0325          GMA = (SB/EM2) - 1.
ISN 0326          EMF = 0.5*EM2*(GMA-1.)
ISN 0327          EMF = GMA*EM2/(TPB*(1.+EM2*EMF))
ISN 0328          DO 91 N = 1,NMX
ISN 0329          W = -DPHI(N)/(TPB*RO(N))
ISN 0330          U = CONST - W*RO(N)
ISN 0331          DB = 57.296*ATAN((W-U*RO(N))/(1.+RO(N)*RO(N)))
ISN 0332          SPR = 1. + EMF*DPHI(N)
ISN 0333          WRITE(6,221) N,RO(N),DPHI(N),W,U,DB,SPR
ISN 0334       91 CONTINUE
ISN 0335      220 FORMAT(///38X,'PERFORMANCE DATA',
              *    ///10X,'N',5X,'RHO(N)',6X,'DPHI(N)',7X,'W/U INF',6X,'U/U INF',
              *    4X,'ARCTAN(UN/WO)',4X,'P02/P01',
              *    /68X,'(DEGREES)',/)
ISN 0336      221 FORMAT(8X,I3,4X,F7.4,4X,1PE10.3,
              *                                 5X,0PF8.4,5X,F8.4,6X,F6.2,9X,F6.3)
ISN 0337          RETURN
            C
            C  PRINT MACH NUMBERS
            C
ISN 0338       96 REL = SQRT(EM2*(1.+A(N)) )
ISN 0339          CF = SB/(2.*EM2)
ISN 0340          DO 92 K = 2,KMXM1
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ISN 0341          DO 93 L = 1,LMX
ISN 0342          US(L,K) = UB*FAK(K)*(F(L,K+1,N) - F(L,K-1,N))
ISN 0343          UN(L,K) = REL*(1.+CF*US(L,K))
ISN 0344       93 CONTINUE
ISN 0345       92 CONTINUE
ISN 0346          WRITE(6,290) ITR,ITK,N,RO(N)
ISN 0347      290 FORMAT(1H1,3X,'VALUES OF THE LOCAL MACH NUMBER AFTER ITR = ',I4,
                *    ', ITK = ',I4,'  AT RHO(',I2,') = ',F7.4)
ISN 0348          LA = 1
ISN 0349          LB = 20
ISN 0350          IF(LMX.LT.20) LB = LMX
ISN 0352       97 WRITE(6,291) (LOP(L),L=LA,LB)
ISN 0353      291 FORMAT(/1X,'K  L=',I2,19I6)
ISN 0354          WRITE(6,293)
ISN 0355      293 FORMAT(/)
ISN 0356          DO 94 K = 2,KMXM1
ISN 0357          WRITE(6,294) K,(UN(L,K),L=LA,LB)
ISN 0358       94 CONTINUE
ISN 0359      294 FORMAT(1X,I2,20F6.2)
ISN 0360          LA = LA + 20
ISN 0361          IF(LA.GT.LMX) GO TO 30
ISN 0363          LB = LB + 20
ISN 0364          IF(LB.GT.LMX) LB = LMX
ISN 0366          GO TO 97
ISN 0367          END
```

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          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
               C
   ISN 0002          SUBROUTINE BVAL(IBC)
   ISN 0003          COMMON ZT(30),XI(60),RO(10),F(30,60,10),A(10),B(10),D(10),E(10),
                    *   EE(30),FF(30),SV(30),DPHI(10),BV(10),US(30,60),UN(30,60),
                    *   DNDS(30,10,2),FAK(60),FHW(60),UTA(30),UTB(30),UTC(30),BBV(30),
                    *   BSA(30),BSB(30),BSC(30),VSA(30),VSB(30),BAV(30),LOP(30)
                    *   ,SONIC(10)
   ISN 0004          COMMON DAF,DBF,DCF,AKLKE,RX,OX,CONST
   ISN 0005          COMMON H,DRO,DZT,TDZ,TPB,XIC,XIB,XID,TDR,AKLK,DRO2,DZT2
   ISN 0006          COMMON OMCU,RHUB,RTIP,R2,RXE,RXH,UA,UB,TRDZ,ERA,ERB,CKLKE,SB,
                    *   BTR,FXB,FXI,EM2,AMA,AMB,DM,DKA,DKR,DQ,DQS,TZ,TTZ,TZS,TRS,TDQ
   ISN 0007          COMMON K,N,IDM,     KMX,LMX,NMX,KMXM1,KMXM2,LMXM1,LMXM2,NMXM1,
                    *   NMXM2,KTEO,IRX,    ITK,ITR,ITRMX,KLEP,ITPR,KUP,KDN
   ISN 0008          COMMON ID(36)
   ISN 0009          DO 1 K = 1,KMX
   ISN 0010          IF(XI(K).GE.XIC) GO TO 2
   ISN 0012        1 CONTINUE
   ISN 0013        2 KLE = K
   ISN 0014          KLEP = KLE - 1
   ISN 0015          K2 = KTEO - 1
               C
               C  KLEP IS THE LAST POINT UPSTREAM OF THE L. E., AND KTEO IS THE FIRST
               C  POINT DOWNSTREAM OF THE T. E.
               C
   ISN 0016          GO TO (100,200),IBC
   ISN 0017      200 DO 20 N = 1,NMX
   ISN 0018          R2 = RO(N)*RO(N)
               C
               C   FOR IBC = 2, TAU = TMAX(R)/C(R).
               C   THE BLADE GEOMETRY USED HERE HAS TMAX = CONSTANT.  BV(3) CONTAINS
               C   TMAX/CAX, BV(2) CONTAINS SMALL A, BV(1) AND BV(4) CONTAIN DELTA CP
               C  AT THE HUB AND TIP, RESPECTIVELY.  DELTA CP ZERO VARIES LINEARLY WITH
               C  RHO BETWEEN THESE TWO VALUES.
               C
   ISN 0019          TAU = BV(3)/SQRT(1.+R2)
   ISN 0020          AOM = BV(2)*OMCU
   ISN 0021          ACP = (BV(4)-BV(1))/(RO(NMX)-RO(1))
   ISN 0022          BCP = (BV(1)-H*BV(4))/(1.-H)
   ISN 0023          DCP = ACP*RO(N) + BCP
   ISN 0024          DO21 K = KLE,K2
   ISN 0025          KB = K - KLEP
   ISN 0026          HCP = DCP/2.
   ISN 0027          IF(XI(K).GT.AOM) GO TO 22
   ISN 0029          DCP2 = HCP
   ISN 0030          DNDS (KB,N,1) =  HCP*XI(K)
   ISN 0031          GO TO 23
   ISN 0032       22 DNDS(KB,N,1) = HCP*(AOM+(XI(K)-AOM)*(1.-0.5*(BV(2)+XI(K)/OMCU))
                    *   /(1.-BV(2)))
   ISN 0033          DCP2 = HCP*(1.-XI(K)/OMCU)/(1.-BV(2))
   ISN 0034       23 TPS = 4.*TAU*(1.-2.*XI(K)/OMCU)
   ISN 0035          DNDS(KB,N,2) = R2*DCP2/(1.+R2)+RO(N)*TPS
   ISN 0036          DPHI(N) = HCP*(AOM + OMCU*(1.-BV(2))/2.)
   ISN 0037       21 CONTINUE
   ISN 0038       20 CONTINUE
   ISN 0039          WRITE(6,220) BV(1),BV(4),BV(2),BV(3)
   ISN 0040      220 FORMAT(//5X,'BLADE LOADING SPECIFIED FOR THIS CASE HAS DELTA CP',
```

```
                    *    ' =',F8.4,' AT THE HUB,',F8.4,' AT THE TIP',
                    *    //10X,'LOADING IS CONSTANT OVER THE FIRST',2PF6.2,' PERCENT',
                    *    ' OF THE CHORD, AND THEN VARIES LINEARLY TO ZERO AT THE ',
                    *    //10X,'TRAILING EDGE',
                    *    //10X,'THICKNESS DISTRIBUTION IS THAT OF A DOUBLE PARABOLIC',
                    *    ' ARC, WITH TMAX/CAX = ',0PF8.4,//)
ISN 0041                 GO TO 10
ISN 0042             100 CONTINUE
                C
                C   THE BLADE GEOMETRY AND ANGLE OF ATTACK ARE
                C   T MAX/CAX = BV(1) + BV(2)/R + BV(3)*R
                C   H MAX/CAX = BV(4) + BV(5)/R + BV(6)*R
                C   ALPHA = BV(7) + BV(8)/R + BV(9)*R
                C   WHERE R = RO(N)/RTIP
                C
ISN 0043                 WRITE(6,210) (BV(JJ),JJ=1,9)
ISN 0044             210 FORMAT(//5X,'BLADE GEOMETRY AND ANGLE OF ATTACK SPECIFIED ARE',
                    *    /20X,'T MAX/CAX = TA + TB/R + TC*R, R = RO(N)/RTIP',
                    *    /25X,'WHERE TA = ',1PE12.3,' TB = ',E12.3,' TC = ',E12.3,
                    *    //20X,'H MAX/CAX = HA + HB/R + HC*R',
                    *    /25X,'WHERE HA = ',1PE12.3,' HB = ',E12.3,' HC = ',E12.3,
                    *    //20X,'ALPHA = AA + AB/R + AC*R   (DEGREES)',
                    *    /25X,'WHERE AA = ',1PE12.3,' AB = ',E12.3,' AC = ',E12.3,//)
ISN 0045                 BV(7) = BV(7)/57.29578
ISN 0046                 BV(8) = BV(8)/57.29578
ISN 0047                 BV(9) = BV(9)/57.29578
ISN 0048                 DO 3 N = 2,NMXM1
ISN 0049                 R = RO(N)/RTIP
ISN 0050                 CBC = 1./SQRT(1.+RO(N)*RO(N))
ISN 0051                 HMXCA = BV(4) + BV(5)/R + BV(6)*R
ISN 0052                 TMXCA = BV(1) + BV(2)/R + BV(3)*R
ISN 0053                 ALPH  = BV(7) + BV(8)/R + BV(9)*R
ISN 0054                 DO 4 K = KLE,K2
ISN 0055                 KB = K - KLEP
ISN 0056                 DNDS(KB,N,1) = (4.*HMXCA + 2.*TMXCA)*CBC*(1.-2.*XI(K)/OMCU) - ALPH
ISN 0057                 DNDS(KB,N,2) = (4.*HMXCA - 2.*TMXCA)*CBC*(1.-2.*XI(K)/OMCU) - ALPH
ISN 0058               4 CONTINUE
ISN 0059               3 CONTINUE
ISN 0060              10 RETURN
ISN 0061                 END
```

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          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                                    SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
                C
  ISN 0002            SUBROUTINE RESID(ISHO)
  ISN 0003            COMMON ZT(30),XI(60),RO(10),F(30,60,10),A(10),B(10),D(10),E(10),
                     *   EE(30),FF(30),SV(30),DPHI(10),BV(10),US(30,60),UN(30,60),
                     *   DNDS(30,10,2),FAK(60),FHW(60),UTA(30),UTB(30),UTC(30),BBV(30),
                     *   BSA(30),BSB(30),BSC(30),VSA(30),VSB(30),BAV(30),LOP(30)
                     *   ,SONIC(10)
  ISN 0004            COMMON DAF,DBF,DCF,AKLKE,RX,OX,CONST
  ISN 0005            COMMON H,DRO,DZT,TDZ,TPB,XIC,XIB,XID,TDR,AKLK,DRO2,DZT2
  ISN 0006            COMMON OMCU,RHUB,RTIP,R2,RXE,RXH,UA,UB,TRDZ,ERA,ERB,CKLKE,SB,
                     *   BTR,FXB,FXI,EM2,AMA,AMB,DM,DKA,DKR,DQ,DQS,TZ,TTZ,TZS,TRS,TDQ
  ISN 0007            COMMON K,N,IDM,     KMX,LMX,NMX,KMXM1,KMXM2,LMXM1,LMXM2,NMXM1,
                     *   NMXM2,KTEO,IRX,    ITK,ITR,ITRMX,KLEP,ITPR,KUP,KDN
  ISN 0008            COMMON ID(36)
  ISN 0009            DIMENSION ERMX(10),KER(10),LER(10)
  ISN 0010            WRITE(6,200) ITR
  ISN 0011        200 FORMAT(//5X,'AFTER ITR = ',I2,' MAXIMUM RESIDUALS ARE',
                     *   //4X,'N',7X,'RESIDUAL',4X,'K',4X,'L',
                     *   7X,'AVG RES',8X,'AVG PHI',8X,'AVG SUM',/)
  ISN 0012            DO 1 N = 2,NMXM1
  ISN 0013            SUM = 0.
  ISN 0014            SNORM = 0.
  ISN 0015            SBOT = 0.
  ISN 0016            KOUNT = 0
  ISN 0017            ERMX(N) = 0.
  ISN 0018            BTR = 1. - EM2*(1. + RO(N)*RO(N))
  ISN 0019            AKLK = D(N)*TZS
  ISN 0020            DKA = -B(N)*TTZ
  ISN 0021            ERA = 1. + 0.5*DRO/RO(N)
  ISN 0022            ERB = 1. - 0.5*DRO/RO(N)
  ISN 0023            DO 4 L = 1,LMX
  ISN 0024            VSA(L) = 0.
  ISN 0025            BSB(L) = 0.
  ISN 0026            BAV(L) = BTR
  ISN 0027          4 CONTINUE
  ISN 0028            DO 2 K = 2,KMXM1
  ISN 0029            AKLKE = AKLK/FAK(K)
  ISN 0030            DKR = TRS*E(N)/FAK(K)
  ISN 0031            DO 3 L = 2,LMXM1
  ISN 0032            BBV(L) = BAV(L)
  ISN 0033            BAV(L) = BTR - SB*FAK(K)*(F(L,K+1,N)-F(L,K-1,N))/TDQ
  ISN 0034            VSB(L) = VSA(L)
  ISN 0035            VSA(L) = 0.
  ISN 0036            IF(BAV(L).GT.0.) GO TO 8
  ISN 0038            VSA(L) = 1.
  ISN 0039          8 BSC(L) = BSB(L)
  ISN 0040            BSB(L) = FHW(K)*(F(L,K+1,N)-F(L,K,N)) -FHW(K-1)*(F(L,K,N)-F(L,
                     *   K-1,N))
  ISN 0041            BSA(L) = FHW(K)*F(L,K+1,N)-2.*FAK(K)*F(L,K,N)+FHW(K-1)*F(L,K-1,N)
  ISN 0042            TMA = BAV(L)*(1.-VSA(L))*BSA(L)+BBV(L)*VSB(L)*BSC(L)
  ISN 0043            TA = ABS(TMA)
  ISN 0044            TMB = A(N)*((1.-VSA(L))*BSA(L)+VSA(L)*BSB(L))
  ISN 0045            TB = ABS(TMB)
  ISN 0046            TMC = -2.*DKA*(-F(L+1,K-1,N)+F(L,K-1,N)+F(L+1,K,N)-2.*F(L,K,N)
                     *   +F(L-1,K,N)+F(L,K+1,N)-F(L-1,K+1,N))
  ISN 0047            TC = ABS(TMC)
```

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ISN 0048          TMD = AKLKE*(F(L+1,K,N)-2.*F(L,K,N)+F(L-1,K,N))
ISN 0049          TD = ABS(TMD)
ISN 0050          TME = DKR*(ERA*F(L,K,N+1)+ERB*F(L,K,N-1)-2.*F(L,K,N))
ISN 0051          TE = ABS(TME)
ISN 0052          TOP = TMA+TMB+TMC+TMD+TME
ISN 0053          BOT = TA +TB +TC +TD +TE
ISN 0054          RES = ABS(TOP)
ISN 0055          SUM = SUM + RES
ISN 0056          SNORM = SNORM + ABS(F(L,K,N))
ISN 0057          SBOT = SBOT + BOT
ISN 0058          KOUNT = KOUNT + 1
ISN 0059          IF(ISHO.EQ.0) GO TO 22
ISN 0061          WRITE(6,205) N,K,L,TMA,TMB,TMC,TMD,TME,TOP,RES
ISN 0062      205 FORMAT(3I5,1P7E13.3)
ISN 0063       22 IF(RES.LT.ERMX(N)) GO TO 3
ISN 0065          KER(N) = K
ISN 0066          LER(N) = L
ISN 0067          ERMX(N) = RES
ISN 0068        3 CONTINUE
ISN 0069        2 CONTINUE
ISN 0070          SUM = SUM/FLOAT(KOUNT)
ISN 0071          SNORM = SNORM/FLOAT(KOUNT)
ISN 0072          SBOT = SBOT/FLOAT(KOUNT)
ISN 0073          WRITE(6,210) N,ERMX(N),KER(N),LER(N),SUM,SNORM,SBOT
ISN 0074        1 CONTINUE
ISN 0075      210 FORMAT(I5,1PE15.3,2I5,3E15.3)
ISN 0076       10 RETURN
ISN 0077          END
```

APPENDIX C

DICTIONARY OF VARIABLES

FORTRAN SYMBOL	ALGEBRAIC EQUIVALENT	DEFINITION, USE, COMMENTS
A(N)	ρ^2	
AE		See Equations (38), (46)
ALF	α_1	Equation (16)
AKL	A_K^L	Equation (30)
AKLK	$\left(\frac{1+\rho^2}{\rho} \frac{\Delta\tau}{\Delta\zeta}\right)^2$	
AKLKE	$\left(\frac{1+\rho^2}{\rho} \frac{\Delta\tau}{\Delta\zeta}\right)^2 / f_K$	
AMA, AMB		Mach number limits for which contours are calculated if IOP=4
B(N)	$-(1+\rho^2)$	
BAV(L)	V_K^L	
BBV	V_{K-1}^L	
BE		See Equations (38), (46)
BGL	$\int_{\rho_H}^{\rho_T} \rho \Delta\phi \, d\rho$	
BKL	B_K^L	Equation (30)
BN	B	Number of blades
BSA(L)	$(1-\mu_K^L)(V_K^L+\rho^2)$	
BSB(L)	$\mu_{K-1}^L V_{K-1}^L$	
BSC(L)	$\rho^2 \mu_K^L$	
BTR	$1 - \frac{W_0^2}{a_\infty^2}$	
BV(I)		Parameters for blade-surface boundary conditions
CALT	C_a/L_T	
CE		See Equations (38), (46)

FORTRAN SYMBOL	ALGEBRAIC EQUIVALENT	DEFINITION, USE, COMMENTS
CKL	C_K^L	Equation (30)
CKLKE	= AKLKE	
CONST	C	See Equation (10)
D(N)	$(1+\rho^2)^2/\rho^2$	
DB	$\tan^{-1}(u_n/W_o)_{z\to\infty}$	Used in Subroutine OUTPUT
DAF,DBF,DCF		See Equation (52)
DKA	$(1+\rho^2)\Delta\tau/2\Delta\zeta$	
DKL	D_K^L	Equation (30)
DKR	$\frac{1+\rho^2}{f_K}\left(\frac{\Delta\tau}{\Delta\rho}\right)^2$	
DM		Mach number interval for which contours are calculated if IOP=4
DNDS(K,N,J)		Surface slopes for IBC=1, loading and thickness parameters for IBC=2, see Equations (17) and (18)
DPHI(N)	$\Delta\phi(\rho)$	
DQ	$\Delta\tau$	
DQS	$(\Delta\tau)^2$	
DRO	$\Delta\rho$	
DRO2	$(\Delta\rho)^2$	
DZT	$\Delta\zeta$	
DZT2	$(\Delta\zeta)^2$	
E(N)	$1+\rho^2$	E(N) is defined to be zero for IDM=2
EMTG	$\omega r_T/a_\infty$	
EMX	M_∞	
EM2	M_∞^2	
ERA	$1+\Delta\rho/2\rho$	

FORTRAN SYMBOL	ALGEBRAIC EQUIVALENT	DEFINITION, USE, COMMENTS
ERB	$1 - \Delta\rho/2\rho$	
F(L,K,N)	$\phi(\zeta, z, \rho)$	
FAK(K)	f_K	
FHW(K)	$f_{K+\frac{1}{2}}$	
FPA,FPB,FPC	${}^N\phi^2_{K-1}, {}^N\phi^2_K, {}^N\phi^2_{K+1}$	
FXB		Used to locate upstream and downstream edges of the grid - see Equation (16)
FXI		
GMA	γ	
H	$h = \rho_H/\rho_T$	
IBC		Indicator for blade-surface boundary conditions
IBN	B	
IDM		= 2, 3, for two, three-dimensional calculations
IOP		Used to select output option
IRPT		Print control
IRX		Equals 2 or 1 if there are or are not any hyperbolic points on the line being relaxed
IRXP		Controls intervals at which residuals are printed
IRXPT		Print control
ISAVE		= 1, 0 if results are, are not to be saved on tape
ISHO		= 1 if residuals are to be shown at every grid point
ISTART		Selects start option
ITK		Counter for iteration in z-direction
ITKMX		Maximum number of iterations in z-direction

FORTRAN SYMBOL	ALGEBRAIC EQUIVALENT	DEFINITION, USE, COMMENTS
ITPR		= 1 if elliptic relaxation factor is tapered, see Equation (A-4)
ITR		Counter for iterations in ρ -direction
ITRMX		Maximum number of iterations in ρ -direction
JPRT		Print control
KDN, KUP		Downstream, upstream limits within which Mach number contours are calculated if IOP=4
KHW	$K + \frac{1}{2}$	
KMX		Number of grid points in the z-direction
KMXM1	KMX - 1	
KMXM2	KMX -2	
KLEP		Last K-station upstream of leading edge
KTEO		First K-station downstream of trailing edge
KW, KWMX		Counters for locations of Mach number contours
LMX		Number of grid points in the ζ -direction
LMXM1	LMX - 1	
LMXM2	LMX -2	
NMX		Number of grid points in the ρ-direction
NMXM1	NMX -1	
NMXM2	NMX - 2	
NPRT		Print control
NPT		Print control
OBM	1 - OBV	
OBV		Relaxation factor for circulation, see Equation (53)

FORTRAN SYMBOL	ALGEBRAIC EQUIVALENT	DEFINITION, USE, COMMENTS
OMCU	$\omega c_a / u_\infty$	Value of Z at trailing edge of blades
OPR	$\sqrt{1+\rho^2}$	
OPT	$\rho \Delta \zeta / \Delta Z)_{OPITMUM}$	See Equation (A-1)
ORT	$\sqrt{1+\rho^2} / \frac{\omega c_a}{u_\infty}$	
OX	$1 - 1/\omega$	
Q	τ	
REL	$\sqrt{M_\infty^2 + (\omega r / a_\infty)^2} = \frac{W_0}{a_\infty}$	Inlet relative Mach number at r
RELSQ	$(W_0 / a_\infty)^2$	
RES	${}^N R_K^L$	See Equation (73)
RHUB	$\rho_H = \omega r_H / u_\infty$	
RO(N)	$\rho = \omega r / u_\infty$	
RTIP	$\rho_T = \omega r_T / u_\infty$	
RX	ω	Relaxation factor, see Equation (33)
RXE		Relaxation factors used at elliptic,
RXH		hyperbolic points
RXM	$1 - \omega$	
R2	ρ^2	
SB	$(\gamma + 1) M_\infty^2$	
SPR	p_{0_2} / p_{0_1}	
SV(L)		Temporary storage for ϕ
TA-TE		See Equations (68)-(72)
TALF	$2 \alpha_1$	See Equation (16)
TDQ	$2 \Delta \tau$	
TDR	$2 \Delta \rho$	
TDZ	$2 \Delta \zeta$	

FORTRAN SYMBOL	ALGEBRAIC EQUIVALENT	DEFINITION, USE, COMMENTS
TMA-TME		Absolute values of TA through TE
TPB	$2\pi / B = \zeta_1$	
TPI	2π	
TRS	$(\Delta\tau / \Delta\rho)^2$	
TRDZ	$2\rho\Delta\zeta$	
TTZ	$\Delta\tau / 2\Delta\zeta$	
TZ	$\Delta\tau / \Delta\zeta$	
TZS	$(\Delta\tau / \Delta\zeta)^2$	
U	$(u/U_\infty)_{z\to\infty}$	Used in Subroutine OUTPUT
UA	ρ	
UB	$[2\Delta\tau\,(1+\rho^2)]^{-1}$	
ULEL	$\frac{u_n}{W_0}\,(\zeta = \frac{2\pi}{B}, z=0, \rho)$	
ULEU	$\frac{u_n}{W_0}\,(\zeta = 0, z=0, \rho)$	
UN	u_n / W_0	
UR	v / U_∞	
US	u_s / W_0	
USCRIT		Sonic value of u_s / W_0
UTA,UTB,UTC		Values of ${}^N\phi^L_K$, ${}^N\phi^L_{K-1}$, ${}^N\phi^L_{K-2}$ from previous z-iteration
UTEL	$\frac{u_n}{W_0}(\zeta = \frac{2\pi}{B}, z = \frac{\omega c_a}{U_\infty}, \rho)$	
UTEU	$\frac{u_n}{W_0}(\zeta = 0, z = \frac{\omega c_a}{U_\infty}, \rho)$	
VSA	μ^L_K	
VSB	μ^L_{K-1}	
W	$(\omega / U_\infty)_{z\to\infty}$	Used in Subroutine OUTPUT
XI (K)	$z = \omega x / U_\infty$	
XIB	z_B	See Equation (16)

FORTRAN SYMBOL	ALGEBRAIC EQUIVALENT	DEFINITION, USE, COMMENTS
XID	$\omega c_a / u_\infty$	
XII	z_I	See Equation (16)
XIMP	z_M	See Equation (16)
ZT(L)	ζ	